Spot fare inspection in urban buses transportation system: strategy and unpredictability under Stackelberg game approach

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1 INTRODUCTION

Fare evasion impacts proof-of-payment (POP) transportation systems, leading to reduced service quality, safety perceptions, economic losses and hindering financial sustainability (Reddy *et al.*, 2011, Cantillo *et al.*, 2022, Barabino *et al.*, 2023). In POP systems, passengers purchase tickets before boarding but may choose to evade fare payment (Barabino *et al.*, 2023).

To mitigate fare evasion, transit authorities can employ two types of fare inspection strategies. *Spot fare inspection strategy* considers the spatial inspection probability distribution over the transportation network under a spot deployment policy. For example, an inspector might be assigned to remain at a key transfer station during peak hours. *Fare inspection patrolling strategy* determines the temporal-spatial inspection probability distribution over the transportation network under a patrolling deployment policy. In this case, an inspector may follow a predetermined route covering multiple stops over time. Regardless of the strategy, the unpredictability of inspections is crucial, since predictable inspections allow evaders to exploit gaps in enforcement. Figure 1 illustrates the different strategies the transit authority has at its disposal.

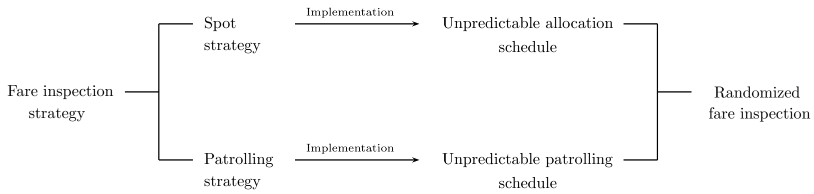

Figure 1 – *Fare inspection strategy implementation, from Escalona* et al. *(2023)*

In this work, the probability distribution of fare inspections over a transportation network is modeled using a Leader–Follower Stackelberg framework (Barabino *et al.*, 2020). The transit authority (*leader*) sets inspection probabilities over the network subject to limited resources while opportunistic passengers (*followers*) respond by evaluating a set of alternative paths and choosing the most convenient for them. The Stackelberg formulation captures the strategic interplay between the authority's inspection strategy and the passengers' best responses based on the steady-state frequencies. This path-based approach to defining the spot strategy leads to a nonlinear optimization problem (NLP) for which a relaxation-based heuristic is proposed. We present an equivalent formulation for the unpredictable allocation schedule, which ensures that for each spot strategy there is a corresponding unpredictable allocation schedule. Two variants of the spot fare inspection system, with selective inspection of buses and mass inspection of passengers on-board are explored. The first one considers that the bus is stopp, andnd all passengers are inspected, as is done in countries like Chile (Delgado *et al.*, 2018). The second one considers the inspection of passengers with the bus in motion.

The main contributions of this work are: (1) It is the first time that a spot fare inspection strategy and their operational implementation in public transportation networks are addressed jointly under a Stackelberg game approach. (2) A mechanism to operationally implement a spot fare inspection strategy is proposed. (3) An efficient approach to generate an unpredictable allocation schedule that takes advantage of the spot inspection strategy is defined. (4) A mechanism for evaluating the time it takes for a spot fare inspection strategy to be operationally implemented is provided.

2 MATHEMATICAL MODELS

2.1 Spot fare inspection strategy formulation

Let us consider an interconnected urban bus network represented by an undirected graph $G = (V, E)$, where V is the set of bus stops (nodes), and E is the set of connections (edges) between stops. The bus system consists of L bus lines, and each line l follows a subset of edges $\mathcal{E}_l \subseteq E$. Nodes and edges may be shared by multiple bus lines.

The transit authority allocates inspection resources to control certain sites within the network each day. Let X represent the set of sites in G that can be inspected. Two inspection systems are considered: (i) edge inspection ($X = E$), where buses are stopped between stations and all onboard passengers are inspected. (ii) district inspection ($X = D$), where inspection teams randomly board moving buses within districts D and inspect passengers without disrupting the bus schedule. In both cases, the goal is to strategically allocate resources to reduce fare evasion.

Opportunistic passengers decide whether to purchase a ticket or evade fare based on the expected cost of their trip, influenced by inspection probabilities. Each passenger type $p \in \mathcal{P}$ is characterized by its origin and destination bus stops, denoted by v_p^+ and v_p^-, respectively. A passenger of type p can either pay a ticket price B for the shortest path or evade fare payment, risking a fine F (where $F \gg B$) if caught. Let $\mathcal{N}_p$ denote the set of k-shortest paths from v_p^+ to v_p^-. The expected value of the amount paid by a passenger is:

$$U_p^o = \min \left\{ B, F \min_{i \in \mathcal{N}_p} \{\mathbb{P}_p^i\} \right\} \quad \forall p \in \mathcal{P}.$$

Thus, the passenger's decision is defined by $\min_{i \in \mathcal{N}_p} \{\mathbb{P}_p^i\}$, and the threshold $c = B/F$.

By expressing the follower's decision through the value function, we can reformulate the bilevel model into a single-level optimization by embedding the passengers' optimal responses in U_p^o.

For an opportunistic passenger of type p on path $i \in \mathcal{N}_p$, the inspection probability is:

$$\mathbb{P}_p^i = 1 - \prod_{(x,l) \in \mathcal{X}_p^i} \left(1 - \mathbb{P}_{xl}^p\right) \quad \forall p \in \mathcal{P}, i \in \mathcal{N}_p, \tag{1}$$

where $\mathcal{X}_p^i$ is the indexed set of sites and bus line pairs that an opportunistic passenger of type p follows and $\mathbb{P}_{xl}^p$ is the probability of an inspection at site (x, l) given by the leader's strategy.

The transit authority's objective is to maximize the expected revenue from ticket sales and fines, formulated as a Stackelberg game. The leader's optimization problem is:

$$x\text{-SSP}: \quad \max_{U,\mathbb{P}} \quad \sum_{p\in\mathcal{P}} \mathbb{E}(d_p^o)\; U_p^o \tag{2}$$

$$\text{s.t:} \quad U_p^o \leq B \qquad \forall p \in \mathcal{P} \tag{3}$$

$$U_p^o \leq F\left(1 - \prod_{(x,l)\in\mathcal{X}_p^i} (1 - f_{xl}\mathbb{P}_x)\right) \qquad \forall p \in \mathcal{P}, i \in \mathcal{N}_p \tag{4}$$

$$\sum_{x\in X} \mathbb{P}_x \leq n \tag{5}$$

$$\mathbb{P}_x \in [0,1] \qquad \forall x \in X, \tag{6}$$

where f_{xl} is the conditional probability and $\mathbb{E}(d_p^o)$ is the expected value for opportunistic passengers. The constraints ensure that payments do not exceed the ticket price B, while the inspection probability limits are defined by the available inspection teams n. The optimal inspection strategy is given by $\{\mathbb{P}_x\}_{x\in X}$.

Let $\mathcal{C}_p^i$ denote the set of inspection probabilities $\mathbb{P}_x$ and utility U_p^o satisfying the constraints for each passenger $p \in \mathcal{P}$ and path $i \in \mathcal{N}_p$. The nonlinear constraints (4) are approximated by the linear form:

$$U_p^o \leq F \sum_{(x,l)\in\mathcal{X}_p^i} f_{xl}\mathbb{P}_x \quad \forall p \in \mathcal{P}, i \in \mathcal{N}_p. \tag{7}$$

This defines the relaxed linear program x-$\mathcal{R}$(SSP), where the optimal objective function $Z^*_{x\text{-}\mathcal{R}(\text{SSP})}$ serves as an upper bound. To compute the lower bound, we calculate the expected payment for each opportunistic passenger:

$$\widehat{U}_p^o = \min_{i\in\mathcal{N}_p}\left\{B, F\left(1 - \prod_{(x,l)\in\mathcal{X}_p^i}(1 - f_{xl}\mathbb{P}_x)\right)\right\}. \tag{8}$$

The lower bound is then $\hat{Z}_x = \sum_{p\in\mathcal{P}} \mathbb{E}(d_p^o)\widehat{U}_p^o$. The optimality gap between the upper and lower bounds measures the quality of the solution.

2.2 An equivalent unpredictable allocation schedule formulation

Let $\mathcal{S}$ denote the set of all possible allocation schedules, each defining the n sites to be controlled in the transportation network during Π. The objective is to determine the probabilities $\pi_s \in [0,1]$ of selecting each schedule $s \in \mathcal{S}$, where the probabilities sum to 1. The probability $\mathbb{P}_x$ that site x is controlled during Π is a convex combination of the allocations, expressed as:

$$\mathbb{P}_x = \sum_{s\in\mathcal{S}} \pi_s Y_{x|s},$$

where $Y_{x|s} = 1$ if x is controlled in schedule s and 0 otherwise. Substituting this into the x-SSP model defines problem x-UAS.

As in the x-SSP model, the x-UAS model is challenging to solve due to the non-convexity of the constraints. We address this using an LP relaxation, denoted as x-$\mathcal{R}$(UAS), which is equivalent to the relaxation of x-SSP.

Given the exponential growth of $\mathcal{S}$ with network size, we propose solving the problem using a Column Generation (CG) algorithm. The algorithm process begins with a subset of schedules $\mathcal{S}_r \subset \mathcal{S}$ and iteratively adds new schedules to improve the solution until optimality is reached.

3 RESULTS

The computational experiments are based on the Berlin urban bus system operating in the city center (Zone AB), where 43 bus lines serving 417 bus stops. Relying on the geographical reference system of the Berlin urban bus system and the route followed by each bus line, we generate G, with $|V| = 417$ and $|E| = 487$. The total number of type of passengers is $|\mathcal{P}| = 86293$. We consider spot fare inspections at 4-6pm, i.e., $\Pi = [16:00, 18:00]$, with 400 000 passengers on average. The set of k-shortest paths for an opportunistic passenger of type p has at most ten alternative paths, i.e., $|\mathcal{N}_p| \leq 10$ for any $p \in \mathcal{P}$. Twenty randomized test problems (test set) were generated, where each of them considers $n \in \{0, ..., 10\}$, leading to a total of 200 instances with common parameters.

We evaluate the performance of the feasible spot strategy and the feasible unpredictable allocation schedule of inspection teams resulting from our CG algorithms, when the sites to be controlled are edges or districts, i.e., $x = e$ or $x = d$. Then, we use Monte Carlo simulation to reproduce the transit authority's daily choice of an inspection team allocation to determine the applicability of the solutions. The most important findings for edge inspections ($x = e$) and district inspections ($x = d$) are summarized in the Table 1.

Table 1 – *Summary of results for edge and district inspections*

Type	**Teams**	**Sites inspected**	**CPU time**	**Gap**	**Evasion rate**	**Days to steady-state**
$x = e$	8 teams	23%	Higher	$\leq 10^{-4}$	$\leq 1\%$	80 days
$x = d$	10 teams	20%	Lower	$\leq 10^{-4}$	$\leq 1\%$	40 days

4 DISCUSSION

This study presents a novel spot fare inspection strategy to mitigate fare evasion in urban bus systems, modeled as a Leader-Follower Stackelberg game. It provides actionable insights into how transit authorities can optimize inspections to effectively reduce fare evasion. The use of a relaxation-based heuristic and CG method enables scalable and efficient implementation, producing near-optimal solutions suitable for large-scale networks. As future work, we plan to incorporate uncertainty in the perception of inspection probabilities by fare evaders.

The results show that both edge and district inspection systems reduce fare evasion, with steady-state evasion rates achievable within 80 days for edge inspections and 40 days for district inspections. Transit authorities can use these insights to determine the optimal number of inspection teams and weigh the trade-offs between strategies based on operational constraints.

Overall, the study offers a practical and comprehensive framework for reducing fare evasion, thereby supporting the financial sustainability of public transportation systems.

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