A deep attention model for solving vehicle routing problems with uncertain parking availability

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1 INTRODUCTION

The field of transportation science and logistics has long grappled with vehicle routing problems (VRPs), a class of combinatorial optimization problems. Since the seminal work by Dantzig & Ramser (1959), numerous VRP variants have been developed to address real-world constraints, including time windows (Kohar et al., 2023), overtime expenses (Mayerle et al., 2020), and fuel consumption (Seyfi et al., 2023). Most prior studies on VRPs utilized simplified networks, where customer nodes are directly connected and travel times are assumed to be constant. However, real road networks include customer nodes, road nodes, and depots, with travel times varying dynamically based on chosen paths and time-dependent link speeds. Therefore, considering real-world road networks and dynamic travel times between customer nodes in VRPs is of more practical significance. Additionally, during customer visits, vehicles often need to find suitable parking at or near each customer's location to facilitate the delivery or pickup of goods. However, parking availability at these locations is often uncertain due to factors such as temporary obstacles, geographical constraints, or capacity limitations. The rapid evolution of urban logistics and increasing congestion in metropolitan areas have made considering uncertain parking availability crucial for improving operational efficiency and customer satisfaction. Thereby, we propose a novel practical Time-Dependent Vehicle Routing Problem (TDVRP) with uncertain parking availability.

Approaches to solving VRPs can be roughly divided into exact techniques, heuristic, and deep learning-based techniques. Exact algorithms can, in principle, obtain optimal solutions, but they are usually problem-dependent and their computational complexity increases exponentially as the problem complexity grows. Compared with exact techniques, heuristic techniques can obtain feasible solutions quickly to various VRPs. However, considering real road networks, time-dependent travel speeds and uncertain parking availability significantly increases the computational complexity of VRPs, making them inefficient. Deep learning-based techniques have emerged as a promising approach to address these challenges. Kool et al. (2019) proposed the Attention Model (AM) to effectively handle various routing problems. Following AM, Guo et al. (2023) developed an AM with dimension-reduction and gate mechanisms (AM-DRGM) to effectively solve the practical TDVRP. To more effectively address our proposed problem, we develop the AM-GA^{3}M, which extends AM-DRGM with node-type-specific and residual attention mechanisms.

2 PROBLEM STATEMENT

A logistics company is tasked with transporting goods from its depot, where the vehicle initiates and concludes its route, to N customer nodes that are geographically scattered in an urban area. A fleet of homogeneous vehicles is deployed to provide transportation services, with all vehicles departing from the depot at time 0 and sequentially delivering goods to customers before returning to the depot. The speed of vehicles varies over time due to factors such as traffic congestion, road conditions, and traffic regulations, which is considered as functions of time and road segments. Additionally, delivery vehicles are required to complete their tasks within a maximum working time limit. Each vehicle has a maximum capacity, and the total weight of goods carried by each vehicle must not exceed this capacity. The location of each customer node i is known, along with its demand q_i. Each customer node has several potential candidate parking lots where delivery vehicles can park. The availability of these parking lots varies over time, and delivery vehicles are only allowed to park and provide services at one and only one candidate parking lots designated for each customer node. If a vehicle parks in a full parking lot, it incurs penalties for illegal parking, and the service time at each customer node is dependent on the selected parking lot by the delivery vehicle. Especially, the parking lot for the depot node is consistently available and coincide with the depot's location. Given these conditions, the objective of the problem is to minimize the total operation cost, including the driving costs, penalty costs for illegal parking, fixed vehicle costs, and overtime pay.

The following assumptions are essential for formulating the mathematical model: (1) Each customer's demand is less than the vehicle's capacity; (2) Each customer is served by exactly one vehicle, only once; (3) Once a vehicle selects a target parking lot, it can only park and provide service at that parking lot.

3 LEARNING MODEL

In the investigated problem, the locations of customer nodes and their corresponding parking lots are often in close proximity, resulting in highly similar node feature information. The AM-DRGM model proposed by (Guo et al., 2023) may struggle to effectively distinguish between customer nodes and their associated parking lots, potentially leading to decreased problem-solving performance. To address this challenge, we propose a Node-Type-Specific Attention Mechanism (NAM), which can significantly improve the model's ability to differentiate between various node types and better capture the complex relationships among nodes. Additionally, since integrating NAM into the AM-DRGM model results in a more complex network structure, ensuring effective information propagation from input to output becomes a significant challenge. To mitigate this issue, we introduce a Residual Attention Mechanism (RAM) (He et al., 2021) to the AM-DRGM model. It can enhance information propagation throughout the model, facilitating the retention of critical features across various layers, thereby substantially improving the model's problem-solving performance.

Fig. 1 shows the architecture of AM-GA^{3}M. The encoder of this model consists of a "N&DR-Trans" block and two identical "Re-Trans" blocks. All these blocks are variants of the "Trans" block proposed by Vaswani et al. (2017), which consists of one Multi-Head Attention (MHA) layer and one linear layer. Both the "N&DR-Trans" and "Re-Trans" blocks modify the "Trans" block by replacing its MHA layer: the former uses an N&DR-MHA layer (a Dimension-reducing MHA layer (Guo et al., 2023) with the NAM), while the latter employs a Re-MHA layer (an MHA layer integrated with the RAM). The encoder processes the input feature information of all nodes, $F = \{f_0, f_1, \cdots, f_{N'}\}$, where $N' + 1$ represents the total number of nodes, to generate node embeddings, effectively capturing complex relationships and characteristics between nodes. For node i, the input feature is denoted as $f_i = \{f_{i,0}, f_{i,1}, \cdots, f_{i,N'}\}$, where each $f_{i,j} = (x_i^h, x_i^v, q_i, T_{i,j,0}, P_{j,0})$ includes node i's horizontal and vertical coordinates, demand, the

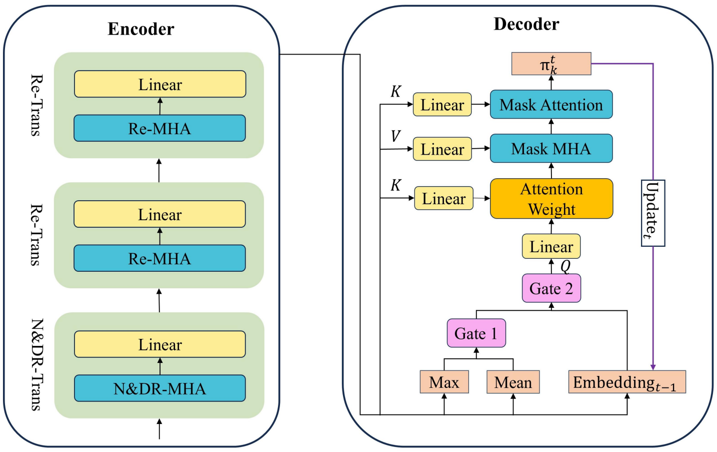

Figure 1 – *Architecture of the AM-GA*^{3}*M*

shortest travel time to node j at time 0, and node j's parking availability probability at time period 0.

The decoder of our model consists of a multi-head attention layer and a mask attention layer. Based on the node embeddings output by the encoder and the index of the previously visited node, it calculates the probability of each node being selected, thereby choosing the next node to visit using either a greedy or sample strategy. The greedy strategy prioritizes selecting the node with the maximum probability value, whereas the sample strategy assigns higher selection weights to nodes with greater probabilities. The decoder executes multiple decoding operations sequentially until all customer delivery requirements are met and the vehicle completes its route by returning to the depot.

4 COMPUTATIONAL RESULTS

4.1 Experimental setting

Our experiments use a dataset that captures travel speeds across 240 consecutive 2-minute time periods on a realistic urban road network. The network consists of 408 nodes and 1,250 directed edges. To generate each problem instance, we randomly select a depot and N customer nodes from the 408 road nodes. For each customer node, we generate 3 parking lots in its vicinity. The service time for each customer is randomly sampled between 30 and 36 minutes. The parking probabilities for each lot across different time periods are randomly sampled from a uniform distribution. Customer demands are randomly sampled between 1 and 9, and the vehicle capacity is set to 50. For this study, we evaluate our model using three distinct sets of problem instances containing 20, 30, and 35 customers, respectively. In total, we generate 271,120 unique problem instances for each size: the first 256,000 are allocated to the training set, the subsequent 10,000 constitute the test set, and the remaining instances are designated as the validation set. To ensure reproducibility of the experimental outcomes, we employed a fixed random seed for generating the training instances.

4.2 Experimental results

Table 1 shows the performance comparison between our AM-GA^{3}M and three benchmarking models. In our problem setting, each customer can park at their own location and is associated with two additional potential parking lots. For example, “Instances with $N = 20$” represents that there are 20 customer nodes and 40 parking nodes in each instance. Each learning-based model uses either the greedy or the sample strategy to select the next node in solution construction.

For example, "AM-GA^{3}M (greedy)" represents that AM-GA^{3}M with the greedy strategy. We report the mean objective values across 10,000 test instances (i.e., "Obj."), the percentage change of the mean objective value of each model relative to the AM model (i.e. "Percentage Change"), and the number of instances in the test set where the model's solution achieves a better objective value than the AM model's solution (i.e. "Opt. Count"). In addition, the "Time" column in Table 1 shows the average time required to solve each collection of 500 problem instances. Each model is trained and tested on problem instances with the same number of customer and parking nodes.

It can be found from Table 1 that, under the greedy strategy, AM-GA^{3}M consistently outperforms other models across three different instance sizes, achieving performance improvements ranging from 1.10% to 10.18%. The performance gap widens as the problem scale increases. In addition, AM-GA^{3}M usually achieves the highest "Opt. Count", except when compared to AM-DRGM with RAM at $N = 20$. This might be attributed to the smaller problem size, where the less complex architecture of AM-DRGM with RAM could be more efficient in exploring the smaller solution space compared to AM-GA^{3}M. Under the sample strategy, the AM-GA^{3}M shows the best overall performance. For $N = 30$ and $N = 35$, it achieves performance improvements ranging from 1.95% to 11.02% and attains the highest "Opt. Count". At $N = 20$, however, AM-GA^{3}M underperforms AM-DRGM with RAM in both "Obj." and "Opt. Count". This might be attributed to the same reasons discussed above.

Table 1 – *AM-GA^{3}M vs Benchmarking Models*

Model	Instances with $N = 20$				Instances with $N = 30$				Instances with $N = 35$			
	Obj.	Percentage Change	Opt. Count	Time	Obj.	Percentage Change	Opt. Count	Time	Obj.	Percentage Change	Opt. Count	Time
AM(greedy)	199.71	0.00%	0	0.22	294.61	0.00%	0	0.31	337.36	0.00%	0	0.34
AM with RAM (greedy)	198.05	-0.83%	5205	0.22	286.73	-2.67%	5659	0.31	340.04	0.79%	4742	0.34
AM-DRGM with RAM (greedy)	188.50	-5.61%	6531	0.44	278.92	-5.32%	5483	1.61	326.47	-3.23%	6297	5.10
AM-GA^{3}M (greedy)	186.43	-6.65%	5784	0.49	268.77	-8.77%	7863	2.22	303.00	-10.18%	8515	5.97
AM (sample)	140.69	0.00%	0	150.43	229.13	0.00%	0	232.78	265.27	0.00%	0	274.72
AM with RAM (sample)	134.08	-4.70%	6205	151.97	221.28	-3.43%	7548	241.17	265.48	0.00%	4979	278.66
AM-DRGM with RAM (sample)	125.25	-10.97%	7888	296.07	207.94	-9.25%	9165	483.23	255.81	-3.57%	7076	1863.87
AM-GA^{3}M (sample)	131.40	-6.60%	7511	297.09	203.89	-11.02%	9524	506.81	241.35	-9.02%	8781	2324.51

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