A Consensus Fixing Based Heuristic for Liner Shipping Network Design with Stochastic Demands

Mikkel Lassen Johansen*, David Pisinger* and Stefan Ropke*
*Technical University of Denmark (DTU)

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1 Introduction

To reach the sustainability goals in maritime shipping, it is important to design a route network that ensures a good utilization of the resources while ensuring sufficient capacity to satisfy the demand. The network design is complicated by fluctuating demand due to seasonal variation in production and demand. In addition, several events in the recent years have highlighted the importance of building robust supply chains where the liner shipping business is a crucial backbone to many of these. Due the significance of liner shipping with naturally fluctuating demand and disruptive events, we consider the *Liner Shipping Network Design Problem with Uncertain Demand* (LSNDP-UD). As stated in (Johansen *et al.*, 2024), the LSNDP-UD has not been subject to extensive studies and thus motivates further solutions methods.

In our work, we exploit the two-folded nature of LSNDP, since this is consisting of 1) designing a network with the objective of minimizing cost and 2) flowing containers through the network in order to maximize revenue. In this structure, we can consider the LSNDP-UN as a two-stage *Stochastic Optimization Problem* (SOP). Here, the first-stage decisions are to design the network, while the second-stage decisions are to flow the containers in the most efficient way depending on the given demand scenario.

When solving SOPs, (Pisinger, 2024) states that there remains a great deal of unexplored methods involving metaheuristics. For this reason, we present a highly parallel metaheuristic based on consensus fixing: The first-stage decisions are solved using an *Adaptive Large Neighborhood Search* (ALNS) framework while the flow problems are solved using an fast column generation algorithm. We run the method in parallel for varying demand scenarios and in intermediate steps, the scenarios try to reach consensus about fixing first-stage decisions. The process of alternating between heuristic solution (first-stage), column generation (second-stage) and consensus fixing is repeated until we reach our termination criteria. We apply this methodology and report computational results using the real-life instances from LINER-LIB (Brouer *et al.*, 2014).

The main idea in the consensus fixing heuristic is to solve each scenario as an independent problem, and then use some heuristic criteria to reach consensus on the first-stage decision variables across the scenarios. Solving independent scenarios is known from *Scenario Analysis* (SCA) where structures from deterministic solutions can help guide decision-making for stochastic problems (King & Wallace, 2012). Consensus fixing thus falls within the SCA category, however, the novelty of the methodology comes from applying this during the search process. This general methodology was first introduced and successfully implemented in (Pisinger, 2024) on a stochastic price collection TSP. The results were quite promising and the consensus framework was proposed to be a general application tool for solving SOPs. In (Pisinger, 2024), the first-stage decisions were binary variables; should a customer be visited or not. This is not directly applicable for the more complex first-stage decision of LSNDP-UN; designing a network. In this work, we thus aims towards extending the consensus framework for SOPs in order to solve more complex solution structures.

The main contribution of our final work can be summarized as follows: (i) We present a two-stage stochastic formulation of the well-known liner shipping network design problem. (ii) We use consensus fixing to solve the stochastic problem. Several setups for the consensus-fixing heuristic are investigated. (iii) We apply ALNS to design routes and a fast column generation algorithm to flow containers through a network. (iv) Detailed computational results are presented for all LINER-LIB instances.

2 Methodology

The origin of consensus fixing is inspired by voting theory where the goal is to reach consent on decisions amongst individuals with deviating objectives (Colomer, 2016). In this setup, we need to make decisions with the greatest impact (or *polarity*) for the individuals (scenarios) in each consensus iteration. In order to introduce the *polarity* measure for the LSNDP-UD, we need to present the voting system.

In our current work, we have relaxed the first-stage consensus decisions of the LSNDP model x to binary variables. The first-stage problem remains the construction of a liner network, however, whenever evaluating consensus, we only consider if a network should traverse a specific arc or not. Assume that we have solved (heuristically or optimally) the SOP corresponding to each scenario $s \in S$, meaning that we have an objective value z_s and a solution vector $\{x_{si}\}$. Let $i \in N_1$ denote the set of all first-stage decision variables and $i \in F$ the set of fixed first-stage variables. For each scenario $s \in S$ and each free decision variable $i \in N_1 \setminus F$, we now calculate the *scores* δ_{si0} and δ_{si1} as estimates for fixing the variable x_{si} to the specific value 0 or 1. Furthermore, for each scenario $s \in S$ we have a *scaling factor* f_s, that express how much weight we should give to the given scenario, which can be based on the ranking of the scenario's z_s. Notice how various strategies can be applied for both the *score* and *scaling factor*. Based on this, we can calculate the *polarity* as shown in (1).

$$\Pi_i = \left| \sum_{s \in S} f_s \ \delta_{si0} - \sum_{s \in S} f_s \ \delta_{si1} \right| \tag{1}$$

The variable with largest value of Π_i is then fixed to value of x_{si} having the cheapest score. Hence, if $\delta_{si0} \leq \delta_{si1}$ then we fix $x_i = 0$, and if $\delta_{si0} > \delta_{si1}$ then we fix $x_i = 1$.

Due to the significant discrepancy between our first-stage problem (liner network) and consensus decision (binary choice of using an arc), we have to make some extensions to the consensus framework. Even if we consider (and fix) all binary consensus variables, we are not guaranteed the same solution across all our scenarios. In case, we enforce a particular arc in all liner networks across scenarios, this arc can be represented by a variety of vessel and speed combinations. For this reason, a termination criteria of evaluating all decision variables might not be the most optimal approach when applying the consensus framework to a problem like LSNDP-UD. In addition to this, we have to determine a method for selecting the final solution amongst the $\mid S \mid$ solutions once the termination criteria is met. For both the termination criteria and the solution selection, we wish to develop general methodologies that can be added to the consensus framework. An overview of the proposed consensus framework for the LSNDP-UD can be seen in Algorithm 1.

Since the main purpose of this work is to test the application of the consensus framework to LSNDP-UD, we need a solution method that is known for producing high quality and consistent results for the LSNDP LINER-LIB instances. For this reason, we have extended the ALNS presented in (Johansen *et al.*, 2024) in order to fit the consensus fixing framework. The ALNS

Algorithm 1: Pseudo-code for LSNDP-UD consensus-fixing heuristic

```
1 ScenarioSolutions = Construct a greedy solution for each scenario s ∈ S;
2 Let the set of fixed first-stage variables be F := ∅;
3 while TerminationCriteria NOT met do
4     Run ALNS for all scenarios s ∈ S solving LSNDP with x_si fixed for i ∈ F;
5     Calculate Π_i for all variables i ∈ N_1 \ F;
6     Select the variable h ∈ N_1 \ F with largest polarity Π_h and fix x_h to the best value;
7     Set F := F ∪ {h};
8 end
9 return ChooseSolution(ScenarioSolutions);
```

will operate as a network improvement framework from which heuristic functions are used to investigate different parts of the solution space by destroying and repairing the considered solution. A weight is assigned to each destroy/repair function, which will be updated according to its performance. These weights will then dictate the likelihood of a given destroy/repair function being selected in the next iteration. The used ALNS contains the following elements:

- Initial Solution - x_{ini}: In order to initialize the ALNS, we apply a backbone flow method (Johansen, 2022)
- Destroy Functions - Ω^-: A total of three destroy functions are used: *RandomCall, LowActivityCall* and *WorstUtilizedRotation*
- Repair Functions - Ω^+: A total of four repair functions are used: *FeederCall, OmissionToRotation, OmissionAsRotation* and *OmissionBackboneFlow*
- *SpeedOptimizer* function: In each ALNS iteration, the canal usage and vessel speed for a rotation is adjusted for the best cost option
- Acceptance Criterion - $Accept(x', x^*, T_{solve})$: Initial tests showed that the developed ALNS gave the best results when only accepting improving solutions. However, to avoid local optima, we will accept a new solution that is 1% worse than the current if no new solution has been accepted for 10% of the total search time T_{solve}
- Function Selection Weights - ρ^-/ρ^+: The initial weights have a value of 1. Let WW denote the reward value given to a function f and γ a decay constant, then the weights are updated as: $\rho_{new}(f) = \gamma \cdot \rho_{old}(f) + (1-\gamma) \cdot WW$. In this work, $WW = 10$ if a new overall best x^* is found, $WW = 5$ if the found solution is better than the current $(c(x', t) < c(x, t))$, $WW = 1$ if the solution is accepted because of the acceptance gap and $WW = 0$ if the solution is not accepted at all. Lastly, the chosen decay constant is $\gamma = 0.99$.

The ALNS in (Johansen *et al.*, 2024) has recently found new best solutions to three of the seven LINER-LIB instances as seen in (Johansen & Røpke, 2024). Notice how the ALNS can be run in parallel in line 4 in Algorithm 1 to speed up the solution process.

3 Results

Preliminary results have demonstrated proof of concept. Next step is to apply the framework for the second-largest instance of LINER-LIB *WorldSmall*. Three different score measures have been made, but no general termination criteria or solution selection have been applied yet. In the presented results, we have thus let the ALNS run for 1100 iterations where it has been subject to

100 consensus fixing rounds. The framework has been tested for between 1-96 different demand scenarios on a setup containing 24 CPU cores. The results of this can be seen in Figure 1.

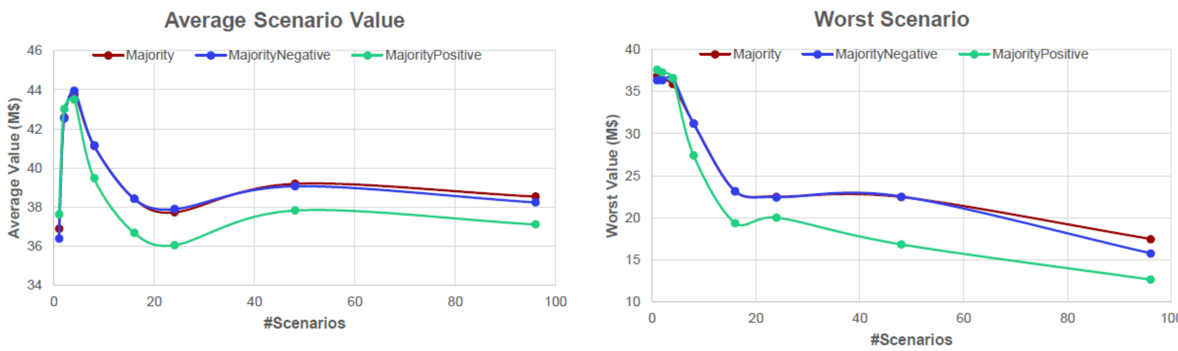

Figure 1 – *Testing three different score measures using between 1-96 scenarios. Left figure: Average profit across the $|S|$ scenario solutions. Right Figure: Worst profit across the $|S|$ scenario solutions*

From the results in Figure 1, we notice clear benefits of including more scenarios to capture the true nature of the LSNDP-UD. In addition to these results, we are quite pleased to observe that the runtime only increases by around 30% when evaluating 24 scenarios compared to a single scenario. Naturally, the runtime increase will be more significant when evaluating more scenarios than we have cores available. However, we still find it promising that the runtime increase with a factor of around 4.5 when considering 96 scenarios compared to just one scenario.

4 Discussion

We believe that our preliminary results are quite promising for applying the consensus fixing framework for a complex problem like the LSNDP-UD. We furthermore expect to present the general methodology framework and tested it fully with different score and scaling functions. At this stage, it will allow us to evaluate the quality of the found solutions and eventually discuss the usage of consensus fixing framework as a general application tool for solving SOPs.

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