UAV Search and Routing Planning In a Disaster Area

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1 INTRODUCTION

This study addresses the problem of optimizing UAV search and routing planning (SRP) to maximize the number of casualties detected in a disaster-affected area within a limited mission duration. We begin by assuming that the number of casualties in different regions is known. The challenge involves determining which regions to visit, in what order, and how much time to spend searching in each region to collect as much expected information about the casualties. The scope of the problem is then extended by incorporating additional real-world complexities, including uncertainties in the number of casualties and coordination of multiple UAVs. The impact of these factors is further explored through a case study based on the major 2023 Turkey-Syria earthquake in which approximately 80,000 people died.

2 METHODOLOGY

Let C_i denote the number of casualties in the region i. Then R_i, the number of casualties found in the region i given a search time of S_i, can be calculated according to equation (1), where $\lambda > 0$ is a UAV detection rate parameter. Figure 1 shows the plot associated with this equation.

$$R_i = C_i(1 - e^{-\lambda S_i}) \tag{1}$$

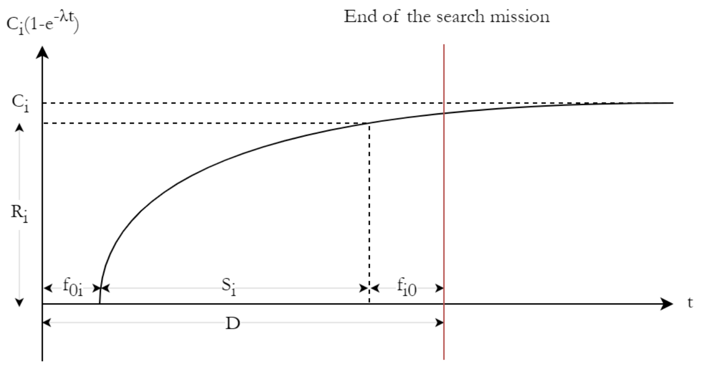


Figure 1 – *Model for detection of casualties.*

It can be seen that equation (1) is increasing at a decreasing rate (concave), reflecting the increasing difficulty of detecting an additional casualty in the target region with the elapsed search time. This exponential form is commonly used in the search literature (e.g., Nie *et al.* (2007)).

To find the optimal solution to the deterministic SRP problem with a single UAV, an exact solution approach has been developed. We derive an equation for the optimal search times for regions along a given route in polynomial time. However, the routing portion of this method is based on brute force, which requires an exponential number of calculations. To solve larger instances, a clustering-based heuristic is developed; it first finds which clusters to visit and in what order, along with the search time in each cluster. The framework is then reapplied to the regions within a cluster. The final step is to determine the entry and exit points of the group.

2.1 Deterministic SRP

We develop a Mixed-Integer Nonlinear Programming (MINLP) model that aims to maximize the total number of casualties detected over all regions. In order to linearize the objective function, each continuous variable S_i is discretized so that it can only take a finite number of possible values, known as its breakpoints. Hence, the resulting Mixed-Integer Linear Programming (MILP) formulation is an inner approximation of the original problem. We create the set of breakpoints using a geometric progression of a variable ratio such that the difference between the number of detected casualties associated with consecutive breakpoints has a constant value. This approach significantly improves the performance of the approximate MILP in comparison to the case where the search time range is simply divided into equal intervals.

2.2 Stochastic SRP

To model the non-deterministic number of casualties, we consider that the region i has C_i casualties, where C_i is a uniform Random Variable (RV) within the range $[C_i^{min}, C_i^{max}]$ with Probability Density Function (PDF) $f(C_i) = \frac{1}{C_i^{max} - C_i^{min}}$ and mean $E(C_i) = \frac{C_i^{min} + C_i^{max}}{2}$. We also assume that the number of casualties in different regions is mutually independent.

Let C_i^t denote the number of casualties found in the region i after spending a search time of duration t in this region. Given C_i, C_i^t is a binomial RV with parameters $n = C_i, p = 1 - e^{-\lambda t}$. Equation (2) calculates the probability of finding k casualties in region i after spending a search time of duration t in this region.

$$\begin{aligned} P(C_i^t = k) &= \int_{C_i^{min}}^{C_i^{max}} P(C_i^t = k|C_i).f(C_i)\, dC_i \\ &= \int_{C_i^{min}}^{C_i^{max}} \binom{n}{k} p^k (1-p)^{n-k}.f(C_i)\, dC_i \end{aligned} \tag{2}$$

The expected value for C_i^t is calculated as:

$$E(C_i^t) = E(E(C_i^t|C_i)) = E(np) = E(C_i)p \tag{3}$$

The variance for the number of casualties in a region equals to $\sigma^2_{C_i^t} = E({C_i^t}^2) - E(C_i^t)^2$, where

$$\begin{aligned} E({C_i^t}^2|C_i) &= \sigma^2_{C_i^t|C_i} + E(C_i^t|C_i)^2 \\ &= np(1-p) + n^2p^2 \end{aligned} \tag{4}$$

$$E({C_i^t}^2) = \int_{C_i^{min}}^{C_i^{max}} E({C_i^t}^2|C_i).f(C_i)\, dC_i \tag{5}$$

The calculated means and variances are fed into the deterministic model to find an initial SRP. The means define the number of casualties in the regions, and the variances are used by a constraint that limits the variance for the total number of casualties in the regions along a given route.

Let path $P : 0-1-2-0$ with search times S_1 and S_2 represent an initial solution. The UAV starts the mission by visiting region 1. Based on Figure 2, after spending some discovery time $t_1^* < S_1$ in the region, comparing the expected number of casualties detected $E(R)$ with the actual number of casualties detected R, we have a more accurate estimate of the number of casualties C_1 and can update the initial solution accordingly. This procedure is repeated each time the UAV flies to a new region.

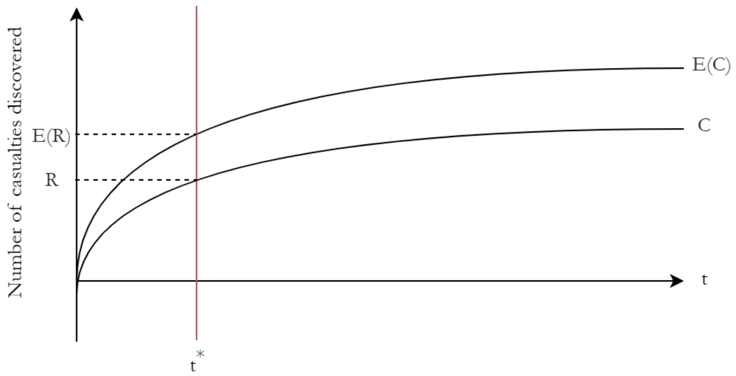


Figure 2 – $E(R)$ *vs* R.

It should be noted that $E(C_i^t)$ and $\sigma^2_{C_i^t}$ not only contribute to finding the initial solution but can also be used to calculate the appropriate discovery times. Equation (6) determines the minimum t^* for a region such that the gap between the estimated and the actual number of casualties in the region is ensured to be below a certain threshold α.

$$t^* = \frac{1}{\lambda}.ln(\frac{a+d+x-c-b}{x+d-b}) \tag{6}$$

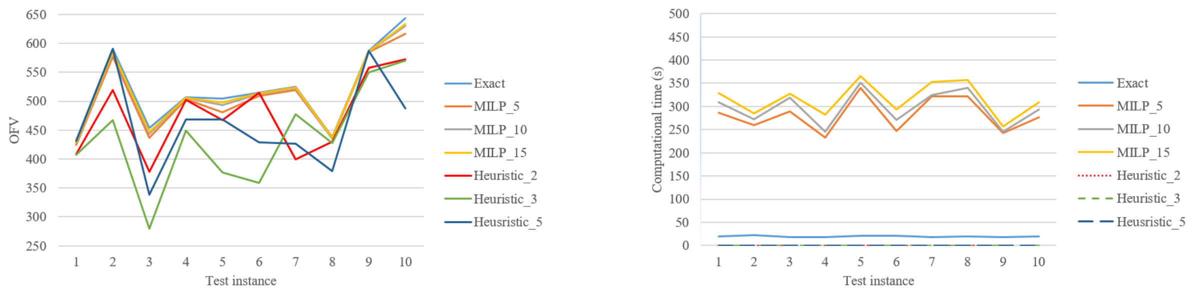
where, $a = \frac{C_i^{max2}}{2}$, $b = \frac{C_i^{max3}}{3}$, $c = \frac{C_i^{min2}}{2}$, $d = \frac{C_i^{min3}}{3}$, $x = (1+\alpha^2)(C_i^{max} - C_i^{min})(\frac{C_i^{max}+C_i^{min}}{2})^2$

3 RESULTS

We performed a series of computational experiments to evaluate the efficiency of the exact method, the approximate MILP, and the clustering heuristic. For this purpose, 10 randomly generated test instances of size 10 are solved using the exact method, the MILP formulation with 5, 10, and 15 breakpoints, and the heuristic algorithm with 2, 3, and 5 clusters. In these cases, search regions are uniformly distributed across a plane with x and y coordinates ranging from 0 to 100. Figure 3 depicts the efficiency of the proposed solution methods for the generated test instances. As expected, using more breakpoints results in better solutions and faster computational times for the MILP problem. In terms of OFV, the improvements are often negligible; however, there are more meaningful improvements as well (e.g., a gap of 0. 23% vs. 3. 43% between MILP_5 and MILP_15 in test instances 9 and 5, respectively). It can be seen that the MILP model is much slower than the exact approach with the average computational times of 281.81, 297.09, and 316.12 seconds for MILP_5, 10, and 15, respectively vs. the average computational time of 19.72 seconds for the exact method. Compared to the other solution methods, more fluctuations appear in the computational time of the MILP model across different test instances of the same size.

The heuristic algorithm is the fastest method that can provide a solution to all instances of problems in less than a second. According to Figure 3a, the number of clusters can significantly affect the quality of the heuristic solution with a minimum gap of 0. 31% and the maximum gap of 30.23% between the corresponding OFVs. However, this parameter does not have a consistent effect on heuristic performance, as its increase might result in an improvement or decline in OFV. We also conduct an experiment that shows variability in the heuristic's performance depending on the distribution of target regions. Solving ten instances each with different region distributions, we find that while the best gap always rests at zero, the number of optimally solved instances, the worst gap, and the average gap considerably improve as the distribution of the target regions moves from entirely uniform to highly clustered. Finally, to determine the time limits of the proposed solution methods, problems of different sizes are solved. According to the results, the exact method can solve up to 14 regions optimally in a reasonable time, whereas the MILP model struggles with instances with more than 10 regions. For the heuristic algorithm, applying the exact approach in parallel within clusters enables it to solve problems of 200 size in less than three hours.

The developed methods have been applied to a case study based on the Turkey-Syria 2023 earthquake, using available data on earthquake characteristics and casualties.



(a) *OFV* (b) *Computational time*

Figure 3 – *Performance of different solution methods for instances of the size 10.*

4 CONTRIBUTION AND NEXT STEPS

Our contributions include: (1) Proposing a MILP model to address the UAV SRP problem with discrete search times, (2) Developing an exact approach to solve the UAV SRP problem with continuous search times, (3) Adapting a clustering heuristic method to solve large-sized problem instances, (4) Extending the scope of the problem by incorporating additional real-world complexities, including uncertainties in the number of casualties and coordination of multiple UAVs, and (5) Presenting a realistic case study to show the successful implementation of this work in humanitarian contexts.

In the next steps of this work, first we need to implement extensive computational testings on the stochastic SRP discussed in Section 2.2. We will also study collaborative stochastic SRPs. This circumstance requires iterative use of the approach discussed in Section 2.2, in that there exists an interplay between new realizations after a certain discovery period, based on which the current solution can be updated.

References

Nie, Xiaofeng, Batta, Rajan, Drury, Colin, & Lin, Li. 2007. Optimal placement of suicide bomber detectors. *Military Operations Research*, 65–78.