Rationally Inattentive Route Choice: A Link-Based Model

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1 INTRODUCTION

In the information-rich society, attention is a finite resource that can easily be consumed by the wealth of available information. The rationally inattentive (RI) choice modelling framework, established in the pioneering works of Sims (2003), quantifies the expenditure of attention as the information cost using the Shannon entropy (Shannon 1948). Later, Matějka and McKay (2015) linked the RI framework with the discrete choice modelling and characterized the RI choice behaviour as a generalized multinomial logit model. Different from the choice behaviours that belong to the random utility model (RUM)'s family (Fosgerau et al. 2013; Mai et al. 2015; Oyama and Hato 2018, 2019), which are driven by exogenous random shocks that are independent of the choice behaviours of the decision maker, Matějka and McKay's work characterizes the choice behaviour that are driven by the endogenous ingredients–the (un)conditional choice probabilities from the decision maker.

Applying the RI concept in transportation, Jiang et al. (2020) and Zhou and Liu (2024) developed the RI framework to address the route choice problem, wherein the traveller acquires information on the traffic conditions so as to optimally choose the routes for their journeys. In these two works, the set of candidate routes is considered as an input, notwithstanding the demanding task of generating all these routes in large transportation networks (Fosgerau et al. 2013). Therefore, the major challenge for the route-based RI choice model is that its scale grows exponentially with respect to the increasing scale of transportation network, which is called curse of dimensionality.

An additional challenge is stated in Matějka and McKay (2015): 'the choice probabilities presented by these generalised multinomial logit model can not always be described by a RUM', which means that there exists a gap of the choice behaviours between Matějka and McKay's family and RUM's family. In this paper, we establish a link-based model for the RI route choice problem so as to address the above mentioned two challenges.

2 A LINK-BASED RI CHOICE MODEL

The link-based RI choice model is an extension of the route-based RI choice model, which leverages the RI choice behaviour on the sequential links to describe the RI choice behaviour on the routes that traverses these links. At each node, the RI traveller acquires information on the traffic conditions (states) of out-going links, then learns the RI link choice behaviour so as to minimize the total cost, including expected route travel costs estimated from link travel costs, information costs on acquiring information on possible link travel costs and expected minimized total cost to the link destinations (expected value function).

We take a simple example in Figure 1 to illustrate the difference on the information acquisition process between route-based and link-based RI choice model. In the route-based RI choice model, in order to learn the route choice behaviour, the RI traveller has to acquire information for all the nine links. The full state is denoted by $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_9) \in \mathbb{R}^9$, which belongs to a finite set $\boldsymbol{\Omega} = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_9$, where ω_k is the realization of travel cost for the the k-th link and Ω_k is the set of possible travel costs. Correspondingly, in the link-based RI choice model, the RI traveller only has to acquire information on states of partial links. For example, when the RI traveller arrives Node 1, he only needs to acquire information on Link $(1,2)$, $(1,5)$, $(1,4)$, the states of these links is denoted by $\boldsymbol{\omega}_1 = (\omega_{1;1}, \omega_{1;2}, \omega_{1;3}) \in \mathbb{R}^3$, which belongs to a finite set $\boldsymbol{\Omega}_1 = \Omega_{1;1} \times \Omega_{1;2} \times \Omega_{1;3}$, where $\omega_{1;k}$ is the realization of travel cost for the k-th link originates from Node 1 and $\Omega_{1;k}$ is the set of possible travel costs.

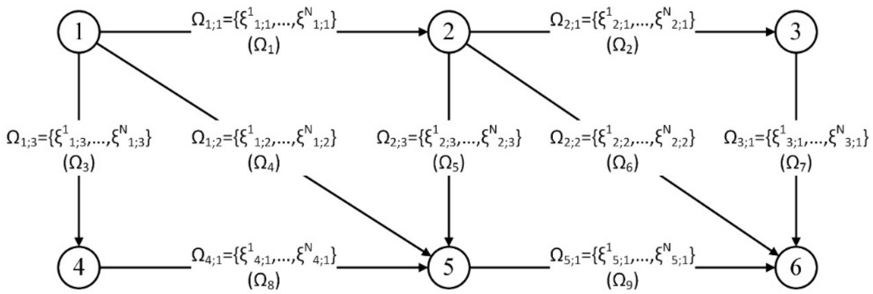


Figure 1 – *A simple network for illustration*

More generally, we consider a transportation network with a directed acyclic graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ and $\mathcal{A}$ are the set of nodes and links respectively. Let the out-going links from node $\mathfrak{h}$ be $\mathcal{A}_{\mathfrak{h}}^{+} = \{(\mathfrak{h}, \mathfrak{h}') \in \mathcal{A} | \mathfrak{h}' \in \mathcal{N}\}$, the out-degree of node $\mathfrak{h}$ be $|\mathcal{A}_{\mathfrak{h}}^{+}|$ and the state of the out-going links be $\boldsymbol{\omega}_{\mathfrak{h}} = (\omega_{\mathfrak{h};1}, ..., \omega_{\mathfrak{h};|\mathcal{A}_{\mathfrak{h}}^{+}|})$ which belongs to a finite set $\boldsymbol{\Omega}_{\mathfrak{h}} = \Omega_{\mathfrak{h};1} \times \cdots \times \Omega_{\mathfrak{h};|\mathcal{A}_{\mathfrak{h}}^{+}|}$.

We describe briefly here the information acquisition processes and the quantification of the amount of information acquired by the travellers. We assume that, prior to commencing the information acquisition process, the RI traveller is only aware of the existence of $|\mathcal{A}_{\mathfrak{h}}^{+}|$ candidate links available to choose from at the outset, which we term as 'background information'. The amount of background information can be measured by the entropy associated with the uniform distribution.

$$\mathcal{H}(\boldsymbol{p} | \Omega_{\mathfrak{h};-1}) = -\sum_{\ell \in \mathcal{A}_{\mathfrak{h}}^{+}} \frac{1}{|\mathcal{A}_{\mathfrak{h}}^{+}|} \ln \frac{1}{|\mathcal{A}_{\mathfrak{h}}^{+}|} = \ln |\mathcal{A}_{\mathfrak{h}}^{+}|,$$

which is independent of the RI route choice behaviour.

Then, the RI traveller acquires habitual information on their unconditional link choice probabilities, $\Omega_{\mathfrak{h};0} = \{p(\ell)\}_{\ell \in \mathcal{A}_{\mathfrak{h}}^{+}}$ from the background, measured by

$$\mathcal{H}(\boldsymbol{p} | \boldsymbol{\Omega}_{\mathfrak{h};-1,0}) = -\sum_{\ell \in \mathcal{A}_{\mathfrak{h}}^{+}} p(\ell) \ln p(\ell).$$

Thus, the amount of acquired habitual information can be measured by

$$\mathcal{I}(\boldsymbol{p}; \Omega_{\mathfrak{h};1} | \Omega_{\mathfrak{h};0}) = \mathcal{H}(\boldsymbol{p} | \Omega_{\mathfrak{h};-1}) - \mathcal{H}(\boldsymbol{p} | \boldsymbol{\Omega}_{\mathfrak{h};-1,0}).$$

For presentation convenience, we denote $\boldsymbol{\Omega}_{\mathfrak{h};1,...,k} = \Omega_{\mathfrak{h};-1} \times \cdots \times \Omega_{\mathfrak{h};k}$ for $k = -1, ..., |\mathcal{A}_{\mathfrak{h}}^{+}|$.

Then, the RI traveller acquires information on the first sub-component of states $\{p(i | \omega_{\mathfrak{h};1})\}$, measured by

$$\mathcal{H}(\boldsymbol{p} | \boldsymbol{\Omega}_{\mathfrak{h};-1,0,1}) = -\sum_{\ell \in \mathcal{A}_{\mathfrak{h}}^{+}} \sum_{\omega_{\mathfrak{h};1} \in \Omega_{\mathfrak{h};1}} g(\omega_{\mathfrak{h};1}) p(\ell | \omega_{\mathfrak{h};1}) \ln p(\ell | \omega_{\mathfrak{h};1}),$$

and the amount of acquired information on the first sub-component $\Omega_{\mathfrak{h};1}$ as:

$$\mathscr{I}(\boldsymbol{p};\Omega_{\mathfrak{h};1}|\boldsymbol{\Omega}_{\mathfrak{h};-1,0})=\mathscr{H}(\boldsymbol{p}|\boldsymbol{\Omega}_{\mathfrak{h};-1,0})-\mathscr{H}(\boldsymbol{p}|\boldsymbol{\Omega}_{\mathfrak{h};-1,0,1}),$$

Repeat the above information acquisition process, the total information cost across all the sub-components of $\boldsymbol{\Omega}_{\mathfrak{h}}$ can be quantified as:

$$C_{\text{info}}(\boldsymbol{p})=\sum_{k=0}^{|\mathcal{A}_{\mathfrak{h}}^{+}|}\lambda_{\mathfrak{h};k}\mathscr{I}(\boldsymbol{p};\Omega_{\mathfrak{h};k}|\boldsymbol{\Omega}_{\mathfrak{h};-1,\ldots,k-1}),$$

where $\lambda_{\mathfrak{h};k}>0$ is the marginal information cost for acquiring information from $\Omega_{\mathfrak{h};k}$.

On the other hand, we dente $c(\ell,\boldsymbol{\omega}_{\mathfrak{h}})$ be the travel cost for link $\ell\in\mathcal{A}_{\mathfrak{h}}^{+}$ in state $\boldsymbol{\omega}_{\mathfrak{h}}$. When the RI choice behaviour $\{p(\ell|\boldsymbol{\omega}_{\mathfrak{h}})\}$ has been learnt, the expected travel cost can be calculated by

$$C_{\mathfrak{h};\text{travel}}(\boldsymbol{p})=\sum_{\ell\in\mathcal{A}_{\mathfrak{h}}^{+}}\sum_{\boldsymbol{\omega}_{\mathfrak{h}}\in\boldsymbol{\Omega}_{\mathfrak{h}}}c\,(\ell,\boldsymbol{\omega}_{\mathfrak{h}})\,g\,(\boldsymbol{\omega}_{\mathfrak{h}})\,p\,(\ell|\boldsymbol{\omega}_{\mathfrak{h}}).$$

The link-based RI choice model can then be formulated as follows:

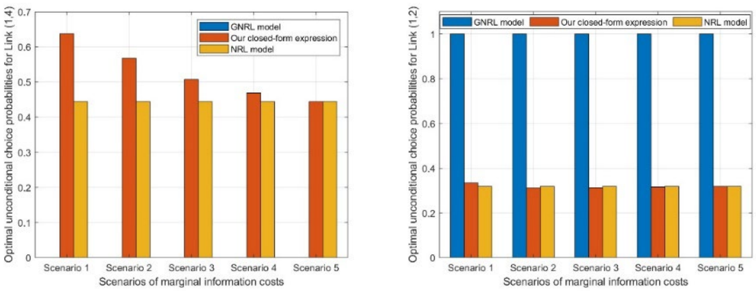
$$\begin{aligned}\min_{\{p(\ell|\boldsymbol{\omega}_{\mathfrak{h}})\}}C(\boldsymbol{p}_{\mathfrak{h}})=&\sum_{\ell\in\mathcal{A}_{\mathfrak{h}}^{+}}\sum_{\boldsymbol{\omega}_{\mathfrak{h}}\in\boldsymbol{\Omega}_{\mathfrak{h}}}c\,(\ell,\boldsymbol{\omega}_{\mathfrak{h}})\,g(\boldsymbol{\omega}_{\mathfrak{h}})\,p(\ell|\boldsymbol{\omega}_{\mathfrak{h}})\\&+\sum_{k=0}^{|\mathcal{A}_{\mathfrak{h}}^{+}|}\lambda_{\mathfrak{h};k}\mathscr{I}(\boldsymbol{p}_{\mathfrak{h}};\Omega_{\mathfrak{h};k}|\boldsymbol{\Omega}_{\mathfrak{h};-1,\ldots,k-1})\\&+\sum_{\ell\in\mathcal{A}_{\mathfrak{h}}^{+}}p(\ell)V(\mathcal{A}_{\mathfrak{h}_{\ell}'}^{+}),\\\textit{subject to }\ 1=&\sum_{\ell\in\mathcal{A}_{\mathfrak{h}}^{+}}p\,(\ell|\boldsymbol{\omega}_{\mathfrak{h}}),\ \text{for all }\boldsymbol{\omega}_{\mathfrak{h}}\in\boldsymbol{\Omega}_{\mathfrak{h}},\\&0\le p(\ell|\boldsymbol{\omega}_{\mathfrak{h}}),\ \text{for all }\ell\in\mathcal{A}_{\mathfrak{h}}^{+},\ \boldsymbol{\omega}_{\mathfrak{h}}\in\boldsymbol{\Omega}_{\mathfrak{h}},\end{aligned}$$

where $V(\mathcal{A}_{\mathfrak{h}_{\ell}'}^{+})\triangleq\min C(\boldsymbol{p}_{\mathfrak{h}_{\ell}'})$ is the value function that represents the optimal total cost for learning the RI choice behaviour among the out-neighbours of link ℓ.

In the full paper, we will provide proof that, by incorporating the background information, the optimal solution of our RI choice model always locates within the interior of the feasible region, i.e., every candidate is assigned a positive choice probability. Using this property, we analytically characterize the closed-form expression of the optimal RI choice behaviour, which resembles a nested recursive logit (NRL) model.

3 A NUMERICAL ILLUSTRATION

Still take Figure 1 as an example. We consider an RI traveler travelling from Node 1 to Node 6, and it is not difficult to note that there are five feasible routes.



(a) (b)

Figure 2 – *Unconditional choice probabilities for partial candidate links in* $\mathcal{A}_{1}^{+}$. *(a) Link (1,2); (b) Link (1,5).*

Figure 2 illustrates the unconditional link choice probabilities at Node 1, which are respectively characterized by our closed-form exprsession, the generlised nested recrusive logit (GNRL) model using the RI choice model in Matějka and McKay (2015) and the NRL model (Fosgerau et al. 2013; Mai et al. 2015). These numerical examples are conducted for fixed marginal information cost $\lambda_{1;3}=20$ and varying $\lambda_{1;0}$ which takes values sequentially from the set $\{4,8,12,16,20\}$ (five scenarios). The remaining marginal information costs take proper values such that $\lambda_{1;0}$, $\lambda_{1;1}$, $\lambda_{1;2}$, $\lambda_{1;3}$ formulate an arithmetic sequence. When $\lambda_{1;0}=0$, our closed-form expression reverts to the GNRL model. As $\lambda_{1;0}$ increases, our closed-form expression gradually approaches the NRL model, and it reverts to the NRL model when $\lambda_{1;0}=\lambda_{1;3}=20$. This consequence implies that incorporating the background information enables our closed-form expression bridge the gap of choice behaviours between Matějka and McKay's family and RUM's family.

Table 1 compares the computational performances for solving the route-based and link-based RI choice model under four different quantities of possible states for each link. The results show that a slight increase in the number of link states leads to a significant increase in the dimension of the route-based RI choice model, consequently leading to a substantial increase in the runtime required for its solution. In comparison, the link-based RI choice model ensures a more manageable increase in its dimension as the quantity of possible states for each link grows. Consequently, this modelling framework maintains a high level of computational efficiency for its solution.

Table 1 – *Runtime for route-based model and link-based model*

Quantity of possible states for each link	Route-based model		Link-based model	
	Dimension (in num.)	Runtime (in sec.)	Dimension (in num.)	Runtime (in sec.)
2	2560	3.06	48	0.02
3	98415	107.98	162	0.07
4	1310720	1661.44	384	0.16

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