Passenger Based Intermodal Connection Optimization of the Italian Passenger Railway Network

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1 INTRODUCTION

The appeal of a railway system for passengers is dependent on several characteristics such as travel and transfer times, or the ticket price. Transfers are of particular importance as they establish connections between an origin and a destination where no direct connection is at hand. They must be feasible, i.e., the time between the arrival of the first train and the departure of the second train must not be too small to establish a connection. Still, it should also not be too large, as transfer waiting time is perceived as inconvenient by passengers. Both the feasibility of connections and the transfer waiting time, and hence the total travel time, can be influenced by adjusting the departure and arrival times of involved trains. In Giacco & Dell'Olmo (2022) an optimization model for the resulting Connection Optimization Problem is investigated for roughly 10,000 daily trains and 2,000 stations in Italy. Computational results indicate potential for increasing the number of connections between 5% for ±1 minute shifts up to 15% for ±8 minute shifts. In this paper, the model is further enhanced by additional constraints to improve the model's accuracy and consider passenger flows given by an OD matrix. The presented enhanced MILP model was evaluated on instances from the Italian passenger railway network. Finally, we will discuss the impact of the extensions of the enhanced model. The Connection Optimization Problem considered here shows some significant similarities to timetabling and line planning problems which is a very well-studied field in railway optimization, see Nachtigall (1999), Liebchen (2008), Caimi *et al.* (2017), or Borndörfer *et al.* (2015) for further insight.

2 THE CONNECTION OPTIMIZATION PROBLEM

Following the notation in Giacco & Dell'Olmo (2022), let I denote the set of trips. Each trip visits a set of stops $S_i \subseteq S$ at stations S. Let $\overline{AT}_i^s, \overline{DT}_i^s$ denote the initially scheduled arrival and departure time of train i at station s. Further let $Inf \leq t_i^s \leq Sup$ be a time shift that results in updated departure and arrival times $AT_i^s = \overline{AT}_i^s + t_i^s$ and $DT_i^s = \overline{DT}_i^s + t_i^s$, respectively. There is a connection between trains i and j at station s if $MCT \leq \Delta_{ij}^s = DT_j^s - AT_i^s \leq MCT + \beta$, where MCT represents the minimum connection time and β the width of the connection time window. The MCT is mostly dependent on the station's topology, i.e., the distance of the involved platforms while β is considered as a parameter of the problem. Let $C \subset S \times I \times I$ denote the set of possible connections. Then the *Connection Optimization Problem* (COP) is to determine time shifts that maximize the total weight of all established connections.

In this problem, two arbitrary connections are interchangeable regardless of their importance for possible passengers. To enable the problem to distinguish between important and non-important connections let $D \in \mathbb{Q}^{|S|\times|S|}$ be an OD-matrix that states the expected passenger demands D. For each pair of origin and destination stations $o, d \in S$ let P_{od} denote a set of paths that connect o and d along trips of I and feasible connections between them. Further, let $\kappa_i \in \mathbb{Q}$ be the maximum passenger capacity of a trip $i \in I$. Then, the *Passenger Connection Optimization Problem* (PCOP) is to determine time shifts such that the weighted sum of the total number of routed passengers along o-d-paths and the number of feasible connections is maximized while the trip's capacities and the demands are not over exceeded.

2.1 A Mixed Integer Linear Programming Formulation to the (P)COP

The (P)COP can be formulated as mixed integer linear programming formulation as follows:

$$\max \; \nu \sum_{(s,i,j)\in C} x^s_{ij} + \mu \sum_{o,d\in S} \sum_{p\in P_{od}} D_{od} y_p \tag{1}$$

$$Inf^s_{ij} \leq \Delta^s_{ij} + (Inf^s_{ij} - MCT)x^s_{ij} \qquad \forall\, (s,i,j) \in C, \tag{2}$$

$$\Delta^s_{ij} + (Sup^s_{ij} - MCT - \beta)x^s_{ij} \leq Sup^s_{ij} \qquad \forall\, (s,i,j) \in C, \tag{3}$$

$$\Delta^s_{ij} = \overline{DT}^s_j + t^s_j - \overline{AT}^s_i - t^s_i \qquad \forall\, (s,i,j) \in C \tag{4}$$

$$\sigma^{s's}_i \leq t^s_i - t^{s'}_i \leq \tau^{s's}_i \qquad \forall\, i \in I,\; s, s' \in S_i, s = succ(s') \tag{5}$$

$$\sum_{p\in P_{od}} D_{od} y_p \;\leq\; D_{od} \qquad \forall\, o, d \in S \tag{6}$$

$$\sum_{p\in P^s_i} D_p y_p \;\leq\; \kappa_i \qquad \forall\, i \in I, s \in S_i \tag{7}$$

$$y_p \leq x^s_{ij} \qquad \forall\, (s,i,j) \in C_p \tag{8}$$

$$t^s_i \in [Inf, Sup] \qquad \forall\, i \in I, s \in S_i \tag{9}$$

$$x^s_{ij} \in \{0,1\} \qquad \forall\, (s,i,j) \in C, \tag{10}$$

$$y_p \in [0,1] \qquad \forall\, p \in P. \tag{11}$$

The MILP formulation contains three types of decision variables. Rational variables t^s_{ij}, that define the shift of the arrival and departure times of each stop of each trip; rational variables y_p that give the fraction of demand D_{od} that is routed along $p \in P_{od}$; as well as binary variables x^s_{ij} deciding whether a connection is feasible or not. The objective function (1) maximizes the weighted sum of the number of established feasible connections and the aggregated routed passenger demand. The constraints (2),(3), and (4) ensure the correct setting of the respective connection feasibility variable, where $Inf^s_{ij} := \overline{DT}^s_j - Inf - \overline{AT}^s_i + Sup$ is a lower and $Sup^s_{ij} := \overline{DT}^s_j + Sup - \overline{AT}^s_i - Inf$ an upper bound on the transfer time from train i to train j at station s. By (5) the deviation of arrival and departure time shifts of consecutive stops of a single trip is bounded to the choice of bounding parameters $\sigma^{s's}_i$ and $\tau^{s's}_i$. The passenger flow is handled by the constraints (6),(7), and (8). Constraints (6) and (7) ensure that the maximum demand for each o-d-pair, respectively the capacity of each trip is not exceeded. (8) makes sure that path variables y_p can only route demand if all connections in C_p are feasible, respectively all connection variables x^s_{ij} along the respective path are equal to one. Finally, variable domains are defined by (9) - (11).

2.2 Computing reasonable o-d-Paths

The PCOP is heavily dependent on the set of feasible passenger paths P that are available to route the given demand. P is computed from a directed graph $G = (N, A)$ which is constructed as follows. The node set N contains artificial origin and destination nodes for each station $s \in S$ as well as departure and arrival nodes for each stop $s \in S_i$ of each trip $i \in I$. The arc set A contains arcs from each artificial departure node to departure nodes of trips at the respective station; arcs connecting the departure node of stop $s' \in S_i$ of trip i with the arrival node of it's succeeding stop $s = succ(s') \in S_i$; arcs connecting an arrival node with the respective departure node of a stop $s \in S_i$; arcs for each possible connection $(s, i, j) \in C$, connecting the arrival nodes of trip i to the departure node of trip j at station s; and arcs connecting arrival nodes of stops $s \in S_i$ to the artificial arrival node of station s. Each arc duration is according to the maximum of the head node's departure or arrival time minus the tail node's departure or arrival time and the minimum connection time between the two stops.

To compute reasonable sets of paths a k-shortest-path-algorithm described in Maristany de las Casas (2024) is used on graph G for each given o-d-pair. The value of k is chosen in dependence on the demand and ranges between 40 for higher and 10 for lower demands.

3 COMPUTATIONAL RESULTS

The MILP formulation modeling the PCOP was evaluated on instances from the Italian passenger railway network. The underlying network contains 3295 stations and roughly 11000 trips of different modes of operation. Ranging from long-distance high-speed train services to urban train services in larger metropolitan areas. The computations considered all regional and urban train services as fixed, while all train arrivals and departures belonging to inter-city and long-distance services are allowed to be shifted. The maximum connection time β equals 30 in all our computations, while the MCT is dependent on the station layout and ranges between 300 and 900 seconds. Moreover, three different parameter settings for $\sigma_i^{s's}$ and $\tau_i^{s's}$ of constraint (5) were considered. For the first setting, which is called *fixed*, we assign $\sigma_i^{s's} = \tau_i^{s's} = 0$ and thus enforce that each stop $s \in S_i$ of a trip is shifted in the same way along the train's route. For the second, so-called *increasing*, setting $\sigma_i^{s's} = 0$ and $\tau_i^{s's} = Sup$, which forces consecutive stops of the same trip to be shifted in a non-decreasing way. Finally, the *unlinked* setting is defined by dropping (5) completely. All computations were performed on Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz with 64 GB of RAM using Cplex (2009) 12.8.0 with an LP-IP gap tolerance of 0.01, a run time limit of 3600 seconds, and a maximum of eight threads in parallel.

Table 1 – *Results for solving the MILP formulation of the COP, i.e., $\nu = 1.0$, $\mu = 0.0$*

	[0,0]		[-1,1]		[-2, 2]		[-3,3]		[-4, 4]		[-5,5]	
	Obj	CPU	Obj	CPU	Obj	CPU	Obj	CPU	Obj	CPU	Obj	CPU
fixed	1.0	0.2	1.01	0.9	1.02	2.6	1.03	110.7	1.04	3473.7	1.05	3601.3
increasing	1.0	0.2	1.02	1.0	1.03	5.0	1.05	37.4	1.06	1001.5	1.07	2148.5
unlinked	1.0	0.2	1.04	1.8	1.07	3.9	1.09	5.9	1.11	12.2	1.13	19.0

Table 1 shows the results with a parameter setting of $\nu = 1.0$ and $\mu = 0.0$, i.e., maximizing the number of feasible connections. Each row of the table refers to either the *fixed*, *increasing*, or *unlinked* setting. Each column headlined with *Obj* marks the relative objective function value, i.e., the objective function value divided by the objective function value obtained by the computation without possible shifting options, i.e., $Inf = 0$ and $Sup = 0$. Columns headlined with CPU solve the runtime in seconds required to solve the respective instance. The three different settings can be interpreted as a sequence with increasing freedom of choice to shift arrival and departure times but with the rising potential of enforcing an infeasible timetable. Especially for the *unlinked* setting even driving times between two stops could become insufficient. Nevertheless, values for the objective function of this setting define a valid bound on the maximum potential of the other settings. It turns out that with an increasing interval of possible arrival and departure time shifts from $[-1, 1]$ to $[-5, 5]$ the maximum number of feasible connections raises to 5, 7, or 13 percent depending on the respective setting. This rise of potential is bought by an increase of the computation time, though this increase is not a crucial one.

Table 2 – *Results for solving the MILP formulation of the PCOP, i.e.,* $\nu = 0.0$*,* $\mu = 1.0$

	[0,0]		[-1,1]		[-2, 2]		[-3,3]		[-4, 4]		[-5,5]	
	Obj	CPU	Obj	CPU	Obj	CPU	Obj	CPU	Obj	CPU	Obj	CPU
fixed	1.0	0.5	1.00	1.0	1.01	1.5	1.01	2.1	1.02	4.7	1.02	10.8
increasing	1.0	0.5	1.00	1.1	1.01	1.6	1.01	2.1	1.02	4.2	1.02	6.5
unlinked	1.0	0.5	1.00	0.8	1.01	0.9	1.01	1.2	1.02	1.9	1.02	2.1

Table 2 shows the results of solely maximizing the number of routed passengers. It turns out that in this case, the gap between the different settings disappears. This might be a result of the fact that some of the connections that could be enforced via the shifting are simply not necessary to route the passenger flow. Additionally, the relative maximum increase compared to the increase seen for the maximum number of connections decreases from 13 to 2 percent this might be related to the fact that the potential increase of routed passengers is bounded by the maximum demand from which large parts could be routed in any case just by direct connections. From the computational point of view, it turns out that all three settings have way shorter computation times compared to the results of Table 1. This is the result of the fact that the flow could be routed in many ways which enlarges the set of optimal solutions within the solution space.

4 CONCLUSION

In this paper, we presented an approach to optimize connections of existing railway timetables taking into account passenger demands, respectively passenger routes. The approach was evaluated on real-life instances from the Italian railway network. Although the approach does not guarantee operationally feasible updated timetables as it does not take the microscopic infrastructure nor laboring constraints into account, it gives insight into the potential of small changes to the timetable. It can be used to point out certain connections that could be beneficial for routing passengers without changing the overall timetable too much. As our computations were limited to shifting only the arrivals and departures of long-distance train services we will further investigate instances where all train types or only regional trains are considered as adjustable.

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