Equitable Workload Allocation in Vehicle Routing Problem with Heterogeneous Drivers

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1 Introduction

Efficient solutions are not necessarily equitable. Their acceptance and implementation may be contingent on a sufficiently fair distribution of resources, responsibilities, and benefits among different stakeholders. In fact, a solution that maximizes the sum of utilities of all the players might not be implementable, because some of the parties might consider it "unfair" as such a solution may be achieved at the expense of some players. In the conventional transportation planning and distribution contexts, the focus has been devoted to either efficiency or equity but rarely to both. In routing problems in the private sector, the most common equity considerations concern internal stakeholders, i.e., the drivers or other personnel providing the service [1]. The aim is to balance the workload allocation to ensure acceptance of operational plans in order to maintain employee satisfaction and morale, reduce overtime, and reduce bottlenecks in resource utilization. Practical examples include balancing the workload of service technicians [2], home healthcare professionals [3], and volunteers [4]. We, therefore, see a new fundamental research gap, which must be addressed and this research seeks to fill it. By focusing on the context of crowdsourced last-mile delivery, this research seeks to develop a novel solution approach for balancing efficiency and equity among crowdsourced drivers. In this context, a variant of the vehicle routing problem (VRP) in which drivers are independent contractors (crowdsourced) will be explored, where the equity indicator is measured based on the workloads assigned to different drivers and consequently benefits earned by them.

2 Problem Statement

The problem is defined on a graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ and $\mathcal{A}$ represent the set of locations and the arcs within graph $\mathcal{G}$, respectively. A group of customers place orders online and expect deliveries at their home locations. Let $\mathcal{D} \subset \mathcal{N}$ be the set of delivery locations of the available orders, where q_j denotes the quantity/size of the delivery at the location of customer $j \in \mathcal{D}$. The travel time between each pair of locations $(j, j'),\ j, j' \in \mathcal{N}$

for which an arc exists in $\mathcal{A}$ is given by $t_{jj'}$ while the transportation cost along an arc (j, j'), denoted by $c_{jj'}$, is proportional to $t_{jj'}$. Online orders are all fulfilled from a single depot o. The deliveries are performed by a set of available occasional drivers, denoted by $\mathcal{V}$. Each driver $v \in \mathcal{V}$ is characterized by their vehicle capacity, Q_v, their origin, s_v, and their destination, e_v, where they are headed after finishing the delivery task. A driver may be willing to dedicate a maximum of $\gamma t_{s_v,e_v}$ minutes to make deliveries, where γ is an indication of the time flexibility of drivers. Drivers are compensated based on the deliveries they make (the prize received by serving a customer), and the mileage traveled.

This problem is a variant of the open vehicle routing problem (OVRP) [5] with a single depot in which the drivers have prefixed ending points. The set of feasible tasks and routing assignment $\mathcal{X}$ incorporates the assignment of delivery tasks to the drivers while specifying the sequence of visits to the customers (routing decision) by each driver. Suppose that $g(\boldsymbol{x})$ returns the mileage cost of a given routing solution $\boldsymbol{x}$. The company aims to maximize workload assignment equity while maintaining a certain level of efficiency. In our definition, a workload assignment is said to be equitable when *profit ratios* of the drivers employed are as close as possible to each other. The profit ratio of driver v is defined as $\rho_v(\boldsymbol{x}) = p_v(\boldsymbol{x})/\overline{p}_v$, where $p_v(\boldsymbol{x})$ is the profit of employed driver v under the workload assignment $\boldsymbol{x}$, and $\overline{p}_v$ is the maximum profit that the employed driver v could possibly earn. In essence, $\overline{p}_v$ is representative of the potentials of driver v, given their personal characteristics including time flexibility, vehicle capacity, and origin and destination locations. The extra cost paid to improve workload assignment equity in a solution is called the *cost of equity*. We assume that the company is willing to accept a set of routes that involve up to $\alpha\%$ increase in the total mileage compensation compared to the least-cost solution if a more equitable profit distribution can be achieved. Therefore, any solution with a mileage cost within the interval $[z^*, (1+\alpha)z^*]$ is considered *efficient* for the company, where z^* corresponds to the least possible mileage compensation given the set of customers $\mathcal{D}$ and the available fleet of drivers $\mathcal{V}$. The routing solution associated with z^* does not necessarily employ all available drivers in $\mathcal{V}$ and may suggest using only a subset $\mathcal{V}^* \subseteq \mathcal{V}$. Once the subset of drivers $\mathcal{V}^*$ associated with z^* is identified, we then aim to improve workload assignment equity in a solution that only involves drivers in $\mathcal{V}^*$. Notice that if a driver is not assigned any delivery tasks, his compensation is assumed to be zero.

Balancing profit ratios has been studied in the context of Nash Social Welfare (`NSW`), which has roots in game theory and pertains, in particular, to bargaining problems. In the solution of a bargaining problem the players, who compete for a higher gain, agree to form a grand coalition [6]. In this study, we aim at maximizing the profit ratio of drivers as the players of a bargaining game using Nash's method. Inspired by the expanded version of `NSW` [7], to equitably assign a set of delivery jobs among drivers in $\mathcal{V}^*$ (i.e., employed drivers), the problem can be formulated as $\max_{\boldsymbol{x}\in\mathcal{X}}\{\prod_{v\in\mathcal{V}^*}\rho_v(\boldsymbol{x}) : g(\boldsymbol{x}) \leq (1+\alpha)z^*\}$.

3 Solution Approach

To compute the minimum total mileage cost (MTMC), i.e., z^*, we propose to employ a branch-and-price approach. Let $\mathcal{R}_v$ be the set of all *feasible* routes for driver $v \in \mathcal{V}$. If a driver is selected by the company, he must first go to the depot to pick up the orders assigned to him and deliver them on the way to his destination. Let binary variable $x_{vr} := 1$ if driver $v \in \mathcal{V}$ is selected and is assigned route $r \in \mathcal{R}_v$; 0 otherwise. Binary parameter δ_{jr} equals 1 iff delivery of order $j \in \mathcal{D}$ is performed on route $r \in \mathcal{R}_v$ of driver $v \in \mathcal{V}$. Let c_{r_v} be the mileage compensation that the company pays to driver $v \in \mathcal{V}$, and p_{r_v} be the profit of driver v from operating *feasible* route $r \in \mathcal{R}_v$. Using these notations, the path-based formulation of the VRP problem can be stated as Model (1).

$$
\begin{aligned}
(\texttt{MTMC}) \quad z^* = \min & \sum_{v\in\mathcal{V}} \sum_{r\in\mathcal{R}_v} c_{r_v} x_{vr} && && \text{(1a)}\\
\text{s.t.} & \sum_{v\in\mathcal{V}} \sum_{r\in\mathcal{R}_v} \delta_{jr} x_{vr} = 1 && j \in \mathcal{D}, && \text{(1b)}\\
& \sum_{r\in\mathcal{R}_v} x_{vr} = 1 && v \in \mathcal{V} && \text{(1c)}\\
& x_{vr} \in \{0,1\} && v \in \mathcal{V}, r \in \mathcal{R}_v, && \text{(1d)}
\end{aligned}
$$

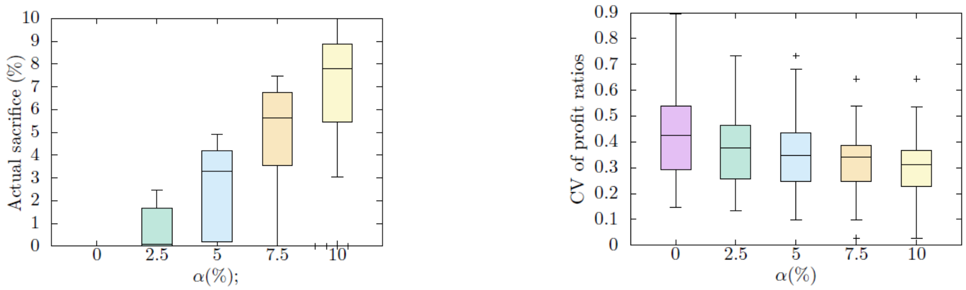
$$
\begin{aligned}
(\texttt{NSW}) \quad \max & \prod_{v\in\mathcal{V}^*} \Big(\sum_{r\in\mathcal{R}_v} p_{r_v} x_{vr} \Big) && && \text{(2a)}\\
\text{s.t.} & \sum_{v\in\mathcal{V}^*} \sum_{r\in\mathcal{R}_v} \delta_{jr} x_{vr} = 1 && j \in \mathcal{D}, && \text{(2b)}\\
& \sum_{r\in\mathcal{R}_v} x_{vr} = 1 && v \in \mathcal{V}^*, && \text{(2c)}\\
& \sum_{v\in\mathcal{V}^*} \sum_{r\in\mathcal{R}_v} c_{r_v} x_{vr} \le (1+\alpha) z^*, && && \text{(2d)}\\
& x_{vr} \in \{0,1\} && v \in \mathcal{V}^*, r \in \mathcal{R}_v, && \text{(2e)}
\end{aligned}
$$

Constraints (1b)-(1d) guarantee that each customer is visited exactly once and each vehicle is assigned to exactly one route. Recall that $\mathcal{V}^* \subseteq \mathcal{V}$ is the subset of vehicles selected by the company as the result of minimizing its total cost through solving the MTMC problem. To compute the NSW solution for improving the workload equity for drivers selected through solving the MTMC problem (referred to as *drivers in the coalition*), we employ a second branch-and-price approach. The path-based formulation of NSW can be stated as Model (2). Observe that NSW is nonlinear. So, in order to employ a branch-and-price approach, it should be linearized. To solve the linearized version of NSW, once again, we can employ column generation. However, this column generation approach is substantially different from the one proposed for the MTMC problem because its subproblems are significantly more challenging than those available in the literature on classical VRPs.

4 Preliminary Results and Conclusion

We generated random instances by adapting instance "R101" of Solomon's VRP benchmark [8] with time windows, with different numbers of customers and drivers with different origins and destinations. We used the demand information in "R101" to set the quantity of deliveries, q_j. Figure 1a shows the actual sacrifice of the company in terms of cost for different values of α. Observe that although the company has the opportunity to sacrifice α% in terms of cost, the NSW may not be able to use all of it due to the discrete nature of the problem. The medians of the boxplots show that in 50% of instances about 75% of α is used. The fact that more equitable solutions are obtained can be seen from Figure

1b where it shows the medians of the Coefficient of Variance (CV) of profit ratios are dropped by about 30% when comparing $\alpha = 0\%$ and $\alpha = 10\%$. Our results show that the proposed method can effectively improve equity in workload allocation and consequently profit distribution among a fleet of heterogeneous drivers.



(a) Actual sacrifice of the company

(b) Equity measure - Coefficient of Variance

Figure 1: Sensitivity analysis for different values of α.

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