A mixed integer programming approach for airspace sector design problem *

Go Nam Lui and Guglielmo Lulli
Lancaster University Management School, LA1 4XY Lancaster (UK)
{g.n.lui, g.lulli}@lancaster.ac.uk

Extended abstract submitted for presentation at the 12th Triennial Symposium on Transportation Analysis conference (TRISTAN XII)
June 22-27, 2025, Okinawa, Japan

October 31, 2024

Keywords: Mixed integer programming, airspace sector design, air traffic management

1 INTRODUCTION

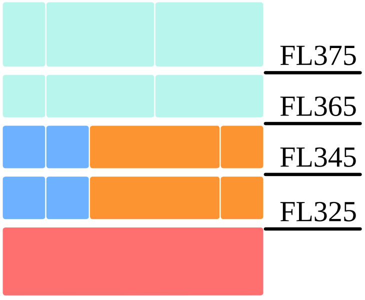
Air traffic controllers (ATCOs) face mounting challenges as they navigate increasingly complex airspace, further exacerbated by adverse weather conditions and the integration of emerging airspace users. To address the challenge of increasing traffic demand and complexity, we aim to make the airspace design and configuration process more efficient taking full advantage of the airspace potential. More specifically, we propose an optimization approach to design airspace sectors. Airspace sectors are three-dimensional volumes of airspace supervised by a team of two controllers, who are responsible for maintaining safe separation and ensuring efficient traffic flow. Our goal is to create a comprehensive catalog of optimized sectors that can be dynamically selected based on varying traffic patterns, weather conditions, and operational requirements. This catalog will serve as a valuable resource for Air Navigation Service Providers (ANSPs) to implement more flexible and efficient airspace management strategies.

In the proposed approach, airspace sectors are obtained by grouping three-dimensional basic volumes, i.e., the atomic portions of the airspace. These basic volumes serve as the fundamental building blocks for constructing optimized sectors. Fig. 1 illustrates the concept of airspace basic volumes using Madrid's airspace as an example. The visualization consists of two complementary views: Fig. 1a shows the vertical stratification of the airspace into distinct flight levels and operational layers, while Fig. 1b provides a top-down view displaying the horizontal distribution of basic volumes across the Madrid Airspace.

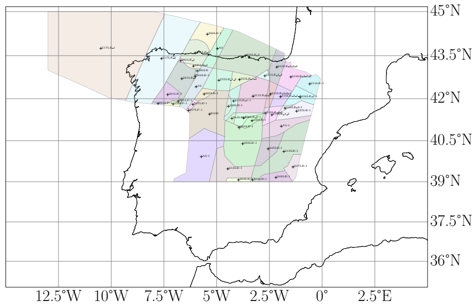
More specifically, we develop a mathematical program that designs multiple sectors simultaneously. The motivation of multiple sector design stems from the challenges that may arise in a single sector design approach. Indeed, it may create an extremely large number of sectors, which might fail to account for system-wide implications and inter-sector relationships. There is no guarantee that feasible configurations can be retrieved from the corresponding sector catalog.

Our proposed approach primarily focuses on two critical operational requirements for sector design: workload balance and air traffic flow convexity. Workload balance refers to the equitable

*The work herein presented is part of the SMARTS project. SMARTS has received funding from the SESAR 3 Joint Undertaking (JU) under grant agreement No 101114686. UK participants in SMARTS receive funding from UK Research and Innovation (UKRI) under the UK government's Horizon Europe funding guarantee [grant numbers 10086651 (Lancaster University)] and 10091277 (NATS). Opinions expressed in this work reflect the authors' views only, and the SESAR 3 JU and UKRI are not responsible for any use that may be made of the information contained herein.



(a) *Abstract vertical profile for the five layers of Madrid airspace.*



(b) *2D projection of basic volumes.*

Figure 1 – *Configuration of Madrid Airspace.*

distribution of aircraft occupancy counts within designated sectors, ensuring a fair allocation of air traffic management responsibilities. Air traffic flow convexity - which can be envisioned as an approximation of the sector convexity property - focuses on aligning sector boundaries parallel to the main traffic flows within each sector. This alignment offers several advantages:

1. It reduces the frequency of aircraft crossing sector boundaries, allowing flights to remain within a single sector for extended periods.
2. It minimizes short crossings and re-entries, lowering the inter-sector traffic flow.

These consideration can reduce workload and communication complexity for the controllers and improves overall airspace efficiency and safety.

Although workload balance and convexity were considered in previous research efforts, they have been used *ex post*, for the evaluation of solutions rather than in the design process (Flener & Pearson, 2013). In this work, our contribution is two-fold. First, we present a Mixed Integer Programming (MIP) model for three-dimensional airspace sector design based on basic volumes aggregation which is the first rigorous mathematical formulation for this problem to the best of our knowledge. Second, we perform numerical experiments on a set of realistic instances based on 105,854 historical flights in August 2024 over the Madrid Area Control Center (ACC) in this preliminary study. Results show the viability of our model for sector design.

2 METHODOLOGY

In this work, we use an undirected graph $G = (V, E)$ to represent the structure of the airspace, i.e., the set of basic volumes and their relation of adjacency. The set E includes an arc $(i{,}j)$, if volumes i and j share a face of the prisms representing the volumes. More specifically the set $E = E_H \bigcup E_V$, where E_H is the set of arc whose endpoints are volumes at the same layers whilst E_V are arcs of adjacent volumes at different (consecutive) layers. To formulate the model, we introduce the following notations:

- $\overline{W}$: maximum capacity (workload) of a sector;
- c_{ij}: horizontal inter-block flow between adjacent volumes i and j (i.e., $(i, j) \in E_H$);
- w_i: workload/occupancy counts in volume i;
- r_{ij}: vertical inter-block flow between adjacent volumes i and j (i.e., $(i, j) \in E_V$).

The decision variable of our formulation is the following:

$$x_{ij} = \begin{cases} 1, & \text{if volume } i \text{ is grouped with volume } j, \\ 0, & \text{otherwise,} \end{cases}$$

and a continuous variable:

$$W = \text{the minimum workload across sectors.}$$

Maximizing the minimum value of the workload W, tend to form an equitable distribution of workload across all the sectors. For the sake of clarity, the index j of variable x_{ij} identifies the sector. Sector j is defined as the set of all volumes i for which $x_{ij} = 1$. We here present the objective function and the basic constraints for our model:

$$\max \quad \alpha \cdot W + (1-\alpha) \cdot [\sum_{i,j:(i,j)\in E_H,k} c_{ij} \cdot (x_{ik} \times x_{jk}) + \sum_{i,j:(i,j)\in E_V,k} r_{ij}(x_{ik} \times x_{jk})]$$

s.t.

$$\sum_{j \in V} x_{ij} = 1 \qquad \forall i \in V. \qquad (1)$$

$$\sum_{i \in V} w_i \cdot x_{ij} \leq \overline{W} \qquad \forall j \in V. \qquad (2)$$

$$W \cdot x_{jj} \leq \sum_{i \in V} w_i \cdot x_{ij} \qquad \forall j \in V. \qquad (3)$$

$$x \in X. \qquad (4)$$

The objective function of the proposed model consists of three components:

- *Workload Balancing*: The first term maximizes the minimum workload across all sectors, which is captured by the decision variable W.

- *Internal Horizontal Traffic Flow*: The second term maximizes the sum of horizontal flow c_{ij} between basic volume i and j within the same sector k.

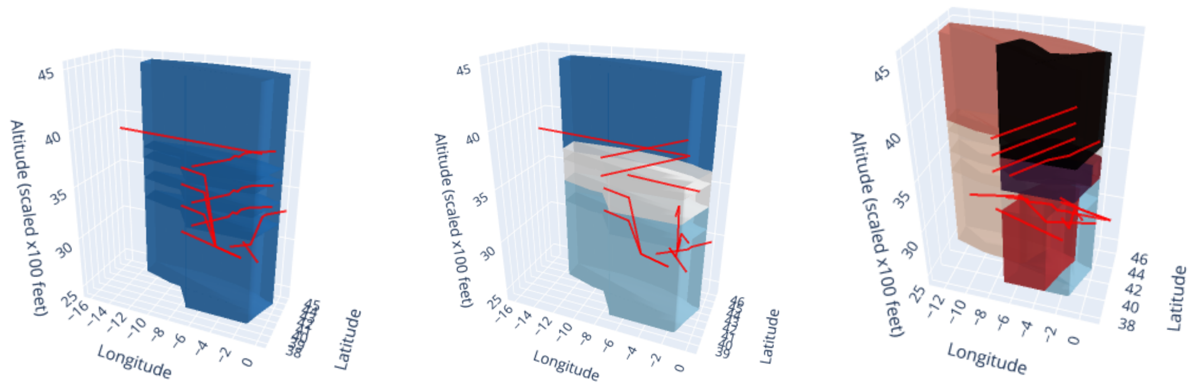
- *Internal Vertical Traffic Flow*: Similar to horizontal traffic flow, This term maximizes the sum of vertical flow r_{ij} between basic volume i and j. The last two terms are equivalent to minimizing the inter-sector traffic flow, as the total exchanged traffic flow is constant for each historical traffic scenario.

Constraints (1) requires each basic volume i to be uniquely allocated to one sector. Constraints (2) imposes that the workload of each sector does not exceed the maximum value. Constraints (3) is used to fix the value of variable W to the minimum workload. We aim to maximize the minimum value W in our objective function to ensure equitable workload distribution. This approach correspond to a *maxmin* approach that is quite common in the equity (or fairness) literature, e.g., Young (1995). Finally, constraint (4) embodies geometrical requirements on the shape of the airspace sectors. Airspace sectors need to be right prisms with a possibly convex polygon base. Indeed, we adopt in our formulation the notion of directional convexity, first proposed in (Holtzman & Halkin, 1966), which imposes each sector to be convex with respect to the main traffic flow inside that sector. Moreover, to ensure horizontal connectivity between basic volumes, we implemented the flow-based formulation, which have been successfully applied in other domains such as political districting (Validi *et al.*, 2022).

3 RESULTS & DISCUSSION

In this section, we present preliminary results from our computational experiments based on a case study of the Madrid Area Control Center (ACC). 105,854 flights used in this study were obtained from the Opensky Network data repository. Our analysis is based on the practical configuration of Madrid ACC, which comprises 5 layers, each containing 50 basic volumes (Fig. 1). The average values of workload and transition are calculated for each hour from 3 a.m. to 22 p.m. based on the raw air traffic data.

The main objective of these computational experiments is the assessment of the operational viability of the proposed approach, specifically, whether the the model designs sectors that are operationally feasible. As an example, Fig. 2 renders the airspace sectors for three traffic scenarios, namely at 5, 6 and 7 am. The red lines depicts the major traffic flows. The interactive figure can be accessed at `https://harrylui1995.github.io/interactive-sector-design-illustration/index.html`, with index representing the considered traffic scenarios. For low traffic periods, the model identifies that the entire airspace can be managed as a single sector. At 6 am the solutions suggests three stacked sectors that share the same projection on the latitude longitude space. At 7 am, with higher traffic levels, nine sectors are designed. Although we do not have guarantee on the optimality of the proposed solutions, upon visual inspection the designed sectors were considered acceptable by practitioners.



(a) *5 am.* (b) *6 am.* (c) *7 am.*

Figure 2 – *3D visualization of the sector design with different traffic scenarios.*

In terms of computational performance, the model is challenging to solve, even for a state of the art commercial solver like Gurobi. For scenarios with a high level of traffic, the model computes solutions with a gap of approximately 7.5% within a one-hour computational time. Given that the design of sectors occurs at a strategic planning phase when time constraints are less stringent, it is acceptable to consider longer computational times to reduce the gap and possibly compute optimal designs. However, we are developing a hyperheuristic approach to further improve computational efficiency while maintaining solution quality.

References

Flener, Pierre, & Pearson, Justin. 2013. Automatic airspace sectorisation: A survey. *arXiv preprint arXiv:1311.0653*.

Holtzman, Jack M, & Halkin, Hubert. 1966. Discretional convexity and the maximum principle for discrete systems. *SIAM Journal on Control*, **4**(2), 263–275.

Validi, Hamidreza, Buchanan, Austin, & Lykhovyd, Eugene. 2022. Imposing contiguity constraints in political districting models. *Operations Research*, **70**(2), 867–892.

Young, H Peyton. 1995. *Equity: in theory and practice*. Princeton University Press.