Information Design for Spatial Resource Allocation

Ozan Candogan[a], Manxi Wu[b,*]

[a] University of Chicago, Booth School of Business

[b] Department of Civil and Environmental Engineering, University of California, Berkeley

[*] Corresponding author

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1 Introduction

In many on-demand mobility and delivery platforms, both jobs and workers (strategic agents) are spatially distributed, often leading to mismatches that cause inefficiencies. Previous research has largely focused on dynamic pricing to address supply-demand mismatches (see Wang & Yang (2019) for a comprehensive review). However, platforms also possess valuable demand data, which they use to guide worker decisions through shared (often partial) information. For instance, both Uber and Lyft use heatmaps to display demand intensity. Despite its practical importance, the role of information design in influencing strategic agent repositioning has received limited attention. Our work introduces a game theory model to study how platforms can leverage demand data to influence worker repositioning and maximize their objectives. We show that, in many relevant cases, a simple monotone partitional information policy is optimal. This policy fully discloses demand states below a lower threshold and above a higher threshold, while concealing demand levels that fall between the two thresholds. We also develop algorithms to determine (near-)optimal monotone partitional structures and apply our model to data from Manhattan's ride-hailing market, demonstrating the effectiveness of the optimal information mechanism.

2 Model and preliminaries

Networks and agents. We consider a network with a set of nodes V and nonatomic agent populations distributed across nodes $i \in V$ with mass vector $m = (m_i)_{i \in V}$. The agent population at each node i decides if they stay at their origin node or reposition to another node j with repositioning cost $c_{ij} \geq 0$. Agents' strategy distribution is $x = (x_{ij})_{i,j \in V}$, where x_{ij} is the mass of agents who move from i to j. A repositioning strategy distribution $x \in X$ is feasible if $\sum_{j \in V} x_{ij} = m_i$ for all $i \in V$ and $x \geq 0$. The distribution of agents induced by x is $q_i = \sum_{j \in V} x_{ji}$ for all $i \in V$. The service price at node i is a linear function $p_i(q_i) = s_i - \beta_i q_i$ for all $i \in V$, where $s_i \geq 0$ is the *market size* at node i, and $\beta_i \geq 0$ is the *price elasticity*. This linear price function can be generalized to piecewise linear function. For every transaction, the platform collects commission with a *fixed* rate $r \in [0, 1]$. Thus, the payoff received by an individual agent is $(1 - r)p_i(q_i)$, and the total commission (i.e. revenue) collected by the platform at i is $rp_i(q_i)q_i$.

One node 0 experiences demand shock that affects the market size s_0. The state of the network is the the realized market sizes $s_0 \in S_0$, where S_0 is a continuous and closed interval of $\mathbb{R}$. The cumulative distribution of the state, referred as *the prior*, is $F : S_0 \to [0, 1]$. The prior F is common knowledge. The platform observes the realization s, but the agents do not.

Platform's information design problem. The platform designs a *public* information provision mechanism $(\mathcal{T}, \pi)$, where $\mathcal{T}$ is the set of possible signal realizations, and $\pi(\cdot|s_0)$ is the probability density function of signal realization given state s_0. The state and the signal sets are both continuous. After observing the state s, the platform generates a signal t according to $\pi(\cdot|s_0)$ and sends the signal to *all* agents. After receiving signal t, agents compute the expected value of the state $\mathbb{E}[s|t]$, and make repositioning decisions based on the received signal, i.e. the strategy distribution $x(t) : \mathcal{T} \to X$. The utility of agents who reposition from i to j equals to the expected payoff received at node j minus the repositioning cost c_{ij}: $u_{ij}(x|t) = (1-r)(\mathbb{E}[s_j|t] - \beta_j q_j(t)) - c_{ij}$. Agents are self-interested in that their repositioning decisions to maximize their expected utility.

Definition 1 *For any $t \in \mathcal{T}$, a strategy profile $x^*(t)$ is a Wardrop equilibrium if*

$$x^*_{ij}(t) > 0, \quad \Rightarrow \quad u_{ij}(x^*|t) \geq u_{ij'}(x^*|t), \quad \forall j, j' \in V, \quad \forall i \in V.$$

The objective function of the platform given state s and signal t is $R(s, q^*(t))$. The goal of the platform is to design the optimal information mechanism $(\mathcal{T}, \pi)$ to maximize their expected objective value given agents' equilibrium strategy:

$$\max_{\mathcal{T},\pi} \int_{s \in S} \int_{t \in \mathcal{T}} R(s, q^*(t)) \pi(t|s) dF(s) dt. \tag{1}$$

Potential function of the repositioning game. To solve (1), we first need to compute Nash equilibrium of the repositioning game given a signal realization. We show that the repositioning game is a potential game, and $x^*(t)$ can be computed as the maximizer of a potential function.

Proposition 1 *For any $t \in \mathcal{T}$, $x^*(t)$ is an optimal solution of the following convex program:*

$$\max_{x \in X} \ (1-r) \sum_{i \in V} \int_0^{q_i(t)} (\mathbb{E}[s_i|t] - \beta_j z) dz - \sum_{i,j \in V} c_{ij} x_{ij}(t), \quad s.t. \quad x(t) \in X. \tag{2}$$

Equilibrium distribution $q^*(t)$ is unique for all $t \in \mathcal{T}$, and depends on t only through $\mathbb{E}[s|t]$.

Proposition 1 allows us to compute equilibrium given an information mechanism, and shows that equilibrium outcome depends on the signal t only through the induced posterior mean $\mathbb{E}[s|t]$.

3 Simple yet optimal information mechanisms

In this section, we focus on identifying the conditions under which optimal information mechanism has a simple partitional structure, referred as monotone partitional. Such mechanism involves partitioning the state space into intervals, and within each interval, the information mechanism either (i) fully reveals the state realization or (ii) only provides information indicating that the realized state falls within that specific interval. To better demonstrate the intuition and practical implications, we first present our results under two assumptions: (i) The platform's objective function is its total revenue $R(s, q^*(t)) = \sum_{i \in V} r q^*_i(t)(s_i - \beta_i q^*_i(t))$; (ii) Before shock happens, all nodes have the same price so that agents have no incentive to move across nodes. We later relax these assumptions. Under assumptions 1-2, we find that monotone partitional information mechanisms is optimal with a variety of practically relevant market size distributions, and the optimal information mechanism can be computed efficiently.

Theorem 1 *Under Assumptions (1) – (2), given any prior distribution F,*

(a) Full information realization is optimal if all nodes have similar market sizes as in (3a).

(b) There exists an optimal monotone partitional information mechanism with thresholds $\inf S_0 \leq \underline{z} \leq \bar{z} \leq \sup S_0$ *that fully reveals states $s_0 \leq \underline{z}$ and $s_0 \geq \bar{z}$, and pools states $s_0 \in [\underline{z}, \bar{z}]$ if market sizes are increasing as in (3b) or decreasing (3c).*

$$\left|\frac{s_i - s_j}{d_i - d_j}\right| \le \frac{1}{1-r}, \quad (3a) \quad \frac{s_i - s_j}{d_i - d_j} > \frac{1}{1-r}, \quad (3b) \quad \frac{s_i - s_j}{d_i - d_j} < -\frac{1}{1-r}, \quad (3c)$$

where d_i is the cost of repositioning from i to shock node 0 along the shortest path. Furthermore, the thresholds $\underline{z}$ and $\bar{z}$ can be computed in $O(|V|)$.

The scenario of similar or monotone market sizes described in Theorem 1 are of practical interests. In practice, the requests of ride-hailing services are often higher in regions close to the central business districts, and decrease in regions far away from the central business districts. In Theorem 1, case (i) corresponds to the scenario where the change of market sizes are small. Case (ii) corresponds to the scenario where the shock happens at a node within the central business district and affect nodes that are outside of the district, which have decreasing market sizes relative to distances. On the other hand, case (iii) corresponds to the scenario where the shock happens at a node that is far away from the central business district, and therefore nodes further away have higher market sizes. Theorem 1 demonstrates that in both cases, the optimal information mechanism fully reveals demand realization when it is above a high threshold $\bar{z}$ or below a low threshold $\underline{z}$, and does not reveal information in between the two thresholds. This corresponds to a very simple and practical information mechanism that sends demand high/low alerts below or above a threshold and does not send any alert in between the thresholds. Theorem 1 shows that such simple information mechanism is optimal regardless of the distribution of demand shock, that is the prior F. The proof of this theorem builds on the equilibrium characterization of our problem and verifying that the equilibrium revenue function $R(q^*)$ satisfies the affine closure condition, which is a sufficient condition for the optimal information mechanism to be monotone partitional introduced in Dworczak & Martini (2019).

Generalization. We drop Assumptions 1 and 2 and allows price function to be piecewise linear instead of being linear. In particular, we generalize our results to objective functions that include (a) Maximizing revenue (b) Maximizing agents' welfare that includes payment minus reposition cost; (c) Minimizing reposition cost. The following theorem summarizes our findings:

Proposition 2 (Informal) *(i) The equilibrium agent distribution q^* and platform's objectives R as in (a) – (c) are piecewise linear functions of the posterior state mean $\mathbb{E}[s_0|t]$. Moreover, we have closed form expressions of $q^*(\mathbb{E}[s_0|t])$ and $R^*(\mathbb{E}[s_0|t])$ for each objective.*

(ii) The optimal information mechanism can be reformulated as a convex program, and the optimal information mechanism is monotone partitional if and only if one set of constraints in the convex program is tight.

(iii) The optimal monotone partitional information mechanism can be computed using dynamic programming and has a fully polynomial-time approximation scheme (FPTAS).

In Proposition 2, (i) shows that equilibrium can be fully characterized in closed form in the general setting, and all four objective functions have piecewise linear structure. Additionally, (ii) shows that the optimal information mechanism can be efficiently computed by a convex program for all three objectives, and the constraint tightness in the convex program can be used to numerically verify whether or not the optimal solution is monotone partitional. This result builds on the piecewise linear objective functions in (i). We further elaborate the dynamic programming approach of approximating the optimal monotone partitional information mechanism in (iii). To achieve an optimal partitional mechanism, we first discretize the state set S with increments of ϵ, and index cutoffs by $\mathcal{I}$. Assuming access to an oracle for evaluating the inverse function (or approximate computations), we define feasible cutoffs $z_i = F^{-1}(\epsilon i)$ for i in $\mathcal{I}$, and calculate probabilities $p(i, i') = F(z_{i'}) - F(z_i)$, and mean objective function $R(i, i') = R\left(\mathbb{E}[s_0|s_0 \in [z_i, z_{i'}])\right)/p(i, i')$ for each interval. Finding an optimal monotone partitional mechanism to maximize the expected objective function reduces to finding the best subset of indexes $\mathcal{I}$. This can be formulated as a dynamic programming with Bellman equation:

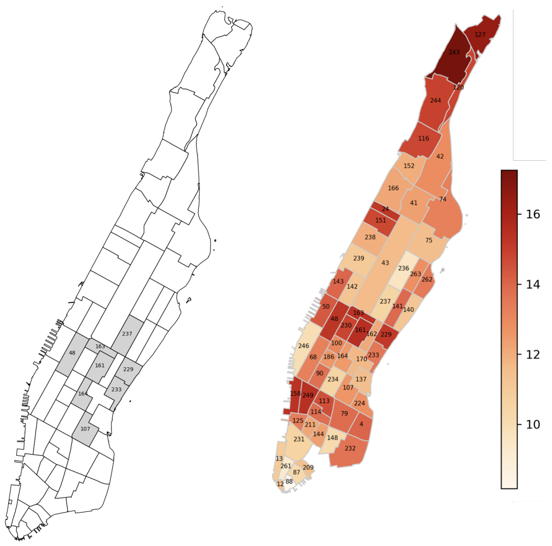
$$V(i_k) = \max_{i \in \mathcal{I} \,|\, i < i_k} \{V(i) + w(i, i_k)\},$$

which yields the optimal mechanism as an FPTAS due to the polynomial-time complexity in $1/\epsilon$.

4 Numerical experiments

We calibrate our model using the For-Hire Vehicle Trip Records in Manhattan from 7-8 pm on all workdays in 2022. We construct a linear regression model to estimate the price function as the relationship between the per-minute passenger payment at each zone and the drivers supply. The demand uncertainty of a zone is the residual of the linear regression.

We select the zones marked in grey in Fig. 1 (left), where the demand shock correlation coefficient exceeds 0.8. We then construct a synthetic demand shock that affects these zones simultaneously and estimate the prior distribution as the cumulative distribution function of residual values from the linear regression. Based on the calibrated model, we compute the optimal information mechanism that maximizes an objective function balancing two goals: (i) revenue maximization R with weight w_R and (ii) minimization of repositioning costs C with weight w_C. We selected one weight tuple, and illustrate the equilibrium price of each zone induced by one signal realization generated by the optimal information mechanism in Fig. 1 (middle). In the table (Fig. 1 Right) we present the range of relative weight parameters, along with the associated total revenue in a 15-minute time slot and the average repositioning time cost per vehicle. We can see that as the relative weight of revenue increases, both the total revenue and the repositioning costs also rise under the optimal information mechanism. This underscores the trade-off the platform faces between increasing revenue and reducing drivers' repositioning costs. Notably, the optimal information mechanisms for all weight parameters follow a monotone partitional structure in our experiment.



w_R	w_C	R^* **($)**	C^* **(min/veh)**
0.0	1.0	13281	3.27
0.1	0.9	13302	3.28
0.2	0.8	13332	3.30
0.3	0.7	13347	3.31
0.4	0.6	13386	3.36
0.5	0.5	13402	3.37
0.6	0.4	13403	3.38
0.7	0.3	13458	3.46
0.8	0.2	13719	4.14
0.9	0.1	13722	4.18
1.0	0.0	13723	4.19

Figure 1 – *(Left) Zones prone to shock in grey. (Middle) Equilibrium price induced by one signal. (Right) Pareto frontier of revenue and reposition cost in 15min under optimal information design.*

References

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