Minimum Multi-Service Fleet Size Problem: Shareability Graph and Network Flow Approach

Dingtong Yang[a], Yubin Liu[a], Hai Wang[b, *], Jinhua Zhao[c], Hamsa Balakrishnan[c]
[a] Singapore – MIT Alliance for Research and Technology Center,
[b] Singapore Management University, [c] Massachusetts Institute of Technology
dingtong.yang@smart.mit.edu, yubin.liu@smart.mit.edu,
haiwang@smu.edu.sg, jinhua@mit.edu, hamsa@mit.edu
[*] Corresponding author

Extended abstract submitted for presentation at the 12th Triennial Symposium on Transportation Analysis conference (TRISTAN XII)
June 22-27, 2025, Okinawa, Japan

Keywords: vehicle-based multi-service, minimum fleet size, ride-hailing, crowdsourced delivery, network graph

1 INTRODUCTION

On-demand, vehicle-based services—such as ride-hailing, food, grocery, and parcel delivery—have become ubiquitous over the past decade. These services can be categorized into four types (Sun et al., 2023): passenger mobility, goods delivery, information acquisition (e.g., probe vehicle for traffic conditions), and mobile server (e.g., vehicle displaying advertisements). Passenger mobility and goods delivery are typically fulfilled by separate fleets, each dedicated to a single service. However, if various services can be pooled and handled simultaneously by a multi-functional fleet while maintaining service quality, the total number of required vehicles and overall vehicle mileage could be significantly reduced. This exciting potential motivates our study to quantify the benefits of the multi-service fleet and address the minimum fleet size problem for multi-service vehicles.

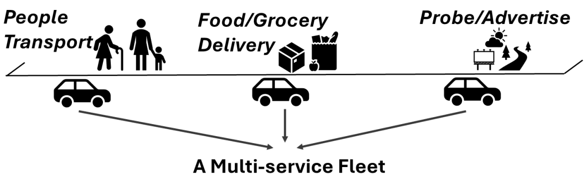

Figure 1 – *Illustration of a Multi-Service Fleet*

In this study, we focus on a multi-service fleet providing three types of vehicle-based on-demand services simultaneously: ride-hailing/ride-sharing (passenger mobility), food/grocery/parcel delivery (goods delivery), and advertising/detecting services (information acquisition/display). While the literature has extensively studied the dispatching and operations of on-demand transportation (Alonso-Mora et al., 2017; Santi et al., 2014; Vazifeh et al., 2018) and crowdsourced delivery(Arslan et al., 2019; Dayarian & Savelsbergh, 2020; Yang et al., 2024), such studies mainly consider single-service fleets. Our paper seeks to address this gap by exploring fleet sharing across different types of services. We present a scalable two-step method to answer the following questions:

- What fleet size reduction can be achieved if different vehicle-based services are fulfilled by a multi-service shared fleet?
- What is the reduction in vehicle kilometers traveled if the multi-service fleet is implemented?
- What is the trade-off between service quality (e.g., waiting time, in-vehicle time) and fleet size?

We address the questions by a two-step approach. First, we construct a shareability hypergraph to indicate whether two or more orders can be served in one vehicle trip. Vertices of the hypergraph represent orders, and hyperedges indicate feasible sharing (i.e. occupy one vehicle and satisfy time constraint) among orders. Second, we propose a network flow model to assign trips obtained from the first step to vehicles, with the objective of minimizing multi-service fleet size or vehicle mileage.

This study contributes to literature in several ways. First, we describe a framework to model vehicle sharing across different types of services in a multi-service fleet. Second, we propose a scalable algorithm that builds a shareability hypergraph between orders and solves the minimum fleet problem using a network flow model. Third, we answer important questions regarding how efficient a multi-service fleet could be in terms of fleet reduction and vehicle miles travelled (VMT) savings.

2 METHODOLOGY

2.1 Constructing two shareability graphs

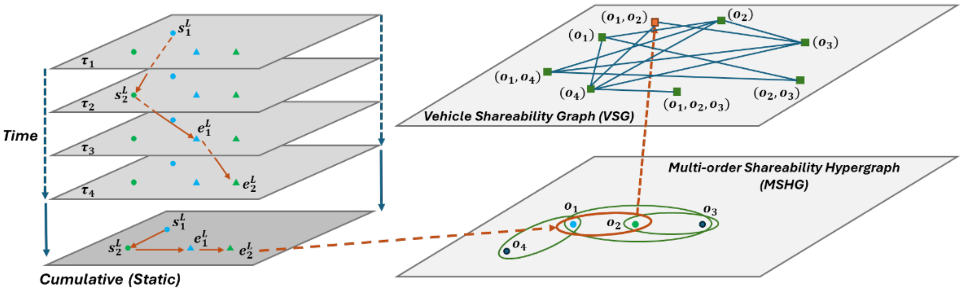

Figure 2 – *Methodology Overview*

A task order *o* is defined as a request for a mobility service, such as ride-hailing, goods delivery, or advertising. Both ride-hailing and delivery orders share the same spatial-temporal structure, each involves transportation from an origin after a specified earliest start time to a destination before a latest arrival time. These orders can be represented as a 4-tuple $(s_i^L, e_i^L, s_i^T, e_i^T)$, where s_i^L and e_i^L denote the starting and ending locations, respectively, and s_i^T and e_i^T indicate the earliest start time and latest end time of the service. In contrast, advertising or information collection services differ in structure. Instead of a single origin-destination pair, they require a vehicle to cover a set of links $(\{l_1, l_2, \ldots\})$ within a time window (s_i^T, e_i^T). To unify the representation across different service types, we can convert these links into a set of origin-destination pairs, allowing all task orders to be expressed in the same 4-tuple format, which enable us to group the orders into trips. **A *vehicle trip* ($\boldsymbol{t}$)** is defined as the movement between locations of a non-idle vehicle with order(s) in service. For example, a trip serves two orders $\{o_1, o_2\}$ in the sequence of $o_1^{pick} \to o_2^{pick} \to o_1^{drop} \to o_2^{drop}$. A valid trip in which no time window constraint is violated indicates the shareability among the orders.

We examine the shareability among tasks to construct a multi-order shareability hypergraph (MSHG), where vertices represent orders and hyperedges represent the shareability of two or more orders by a single vehicle trip (bottom right in Figure 2). Once the MSHG is built, we derive a vehicle shareability graph (VSG) by converting each hyperedge of the MSHG to a node in the VSG (top right in Figure 2). Two nodes in the VSG are connected by an arc if the corresponding trips (originally represented as hyperedges in the MSHG) can be executed sequentially by a single vehicle without time conflicts. Arc weights are the travel cost between the ending location of one trip and the starting location of the next. For example, in top left Figure 2, order o_1 and o_2 share one trip with the sequence of $\left(o_1^{pick}, o_2^{pick}, o_1^{drop}, o_2^{drop}\right)$, visualized by dashed lines in the time expanded network τ_1 to τ_4. In the cumulative static network (bottom left in Figure 2), the trip is shown by the arrows between the pickup and drop-off locations. The same trip also appears as a hyperedge (brown color ellipse) in the MSHG.

Constructing the MSHG has the worst-case computational complexity of $O(n!)$, occurring when each order can share a trip with *any combination* of other orders. However, this scenario is rare in real-world applications due to spatial-temporal constraints and vehicle capacity limits. By restricting the maximum number of orders per trip to a parameter k, the complexity is reduced to $O(n^k)$, allowing us to control computational time by adjusting k. Further reductions in computational time can be achieved through zoning and rolling horizons. Since geographically distant orders are unlikely to share a vehicle, we limit candidate orders for the MSHG to neighboring zones. Similarly, orders with long time separations (e.g., one at noon and another at midnight) are unlikely to share a trip, so we apply a rolling horizon approach with intervals of $(\tau, \tau + 1)$. Algorithm 1 is used to construct MSHG. To adhere to page limits, we present the case of $k = 3$ and the algorithm is extendable to larger k.

Algorithm 1 *Multi-order Shareability Hypergraph Construction*

Initialization: Orders $\{o_1^\tau, o_2^\tau, \ldots, o_{O(\tau)}^\tau\}$ in a horizon $(\tau, \tau + 1)$, maximum shareability $k = 3$.
For $o_i^\tau \in \{o_1^\tau, o_2^\tau, \ldots, o_{O(\tau)}^\tau\}$:
  For $o_j^\tau \in \{o_1^\tau, o_2^\tau, \ldots, o_{O(\tau)}^\tau\}, j \neq i$:
    For $o_m^\tau \in \{o_1^\tau, o_2^\tau, \ldots, o_{O(\tau)}^\tau\}, m \neq j \neq i$:
      Do a *Depth First Search* to permutate $[s_i^L, e_i^L, s_j^L, e_j^L, s_m^L, e_m^L]$
      Check time window: $\tau_{s_i} \geq s_i^T, \tau_{e_i} \leq e_i^T, \tau_{s_j} \geq s_j^T, \tau_{e_j} \leq e_j^T, \tau_{s_m} \geq s_m^T, \tau_{e_m} \leq e_m^T$
      If True time window constraint:
        Return trip t, trip cost c_t
End

In an MSHG, the cost of a hyperedge is the total generalized cost (e.g., vehicle traveled distance or time) of the corresponding trip t. We then represent hyperedges in the MSHG as nodes in the VSG. Each node in the VSG inherits the cost of the corresponding hyperedge in the MSHG as its weight c_t. For each arc from vertex s to t in VSG, if it corresponds to non-overlap hyperedges in the MSHG (i.e., a valid sequence of trips without common orders), we compute its arc cost c_{st} in the VSG as the cost for a vehicle to move from the ending location of predecessor trip s to the starting location of successor trip t. VSG construction constantly has the complexity of $O(n^2)$, where n is the number of trips from MSHG. The steps are similar as Algorithm 1.

2.2 Network flow model for fleet/VMT minimization

With the VSG, we form a network flow model to minimize fleet size and VMT:

$$\min_{x,y} \Theta_1 = \sum_i y_{H,s} \tag{1}$$

$$\min_{x,y} \Theta_2 = \sum_i c_t x_t + \sum_i \sum_j c_{st} y_{st} \tag{2}$$

Subject to:

$$\sum_t a_{it} x_t = 1, \qquad \forall o_i \in Orders, \tag{3}$$

$$x_t + x_s \leq y_{st} + 1, \qquad \forall s, t \in Trips, s \neq t, \tag{4}$$

$$\sum_q y_{qs} - \sum_t y_{st} = 0, \qquad \forall s \in Trips, \tag{5}$$

$$x_t \in \{0, 1\}, \tag{6}$$

$$y_{st} \in \{0, 1\}. \tag{7}$$

Binary decision variables x_t indicate whether a trip (vertex in the VSG) is included; y_{st} indicate whether two trips s and t are served in sequence by the same vehicle. Objective function (1) minimizes fleet size by minimizing the number of unique outflows from a dummy hub (H). Objective function (2) minimizes the total cost, which is the summation of the cost for selected trips (vertex in the VSG) and the cost of connecting trips (edges in the VSG). Eqn. (3) ensures that every order is served by one trip. Constraint (4) indicates that the trip connecting cost is only effective when two consecutive trips are served in sequence by the same vehicle. Eqn. (5) is the vehicle flow conservation constraint.

3 CASE STUDY AND PRELIMINARY RESULTS

We conduct a real-world, city-scale case study in the context of Singapore. We simulate 2,000 ride-hailing orders, 1,000 food/grocery delivery orders, and 230 detection/advertising tasks over an 8-hour period (6:00 am to 2:00 pm) distributed in Singapore. Figure 3 compares the minimum fleet size and total vehicle kilometers traveled for serving all tasks. Subplot (a) demonstrates that, with a multi-service fleet, the fleet size can be reduced by 8% to 19% compared with a single-service fleet, with greater reductions in peak periods than non-peak periods. Total vehicle kilometers traveled exhibits a similar trend: The multi-service fleet reduces vehicle kilometers by 15% to 30%.

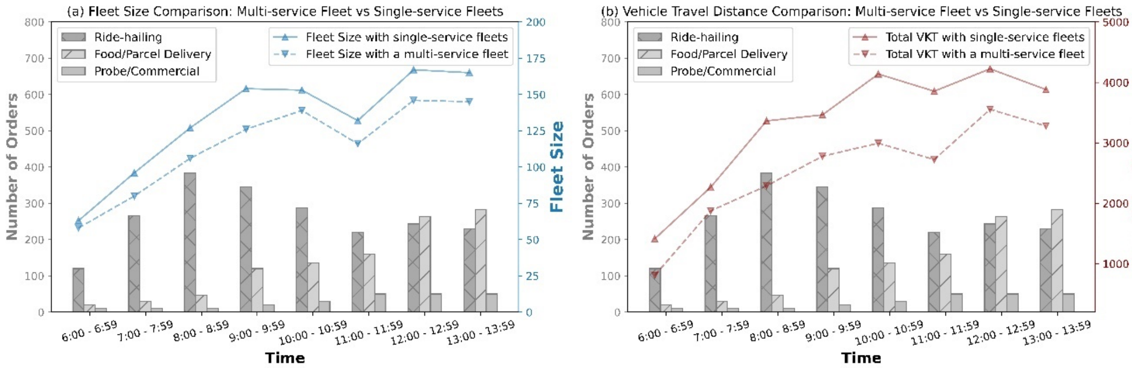

Figure 3 – *Comparison of Minimum Fleet Size and Vehicle Kilometers*

4 DISCUSSION

In this study, we focus on the minimum fleet size problem for a multi-service fleet serving different types of vehicle-based services. Algorithmically, we extend the concept of shareability networks to hypergraph and solve the problem using a network flow model. Computationally, constructing an MSHG with a larger maximum allowable number of tasks in a trip and larger gaps between starting times of orders expands the solution space and improves the solution quality, which comes at the cost of computational efficiency. The case study demonstrates that a multi-service fleet could significantly reduce both the fleet size and vehicle kilometers traveled. A critical factor that affects the efficiency of the multi-service fleet is the customer's tolerance for delay and detour, which are heterogeneous across services and customers. In the full manuscript, we will provide a completed problem statement, detailed descriptions of the construction and modeling of an MSHG, VSG, and network flow problem, and rich numerical experiments with discussions, insights, and implications.

5 REFERENCES

Alonso-Mora, J., Samaranayake, S., Wallar, A., Frazzoli, E., & Rus, D. (2017). On-demand high-capacity ride-sharing via dynamic trip-vehicle assignment. *Proceedings of the National Academy of Sciences*, *114*(3), 462–467.

Arslan, A. M., Agatz, N., Kroon, L., & Zuidwijk, R. (2019). Crowdsourced delivery—a dynamic pickup and delivery problem with ad hoc drivers. *Transportation Science*, *53*(1), 222–235.

Dayarian, I., & Savelsbergh, M. (2020). Crowdshipping and Same-day Delivery: Employing In-store Customers to Deliver Online Orders. *Production and Operations Management*, *29*(9), 2153–2174.

Santi, P., Resta, G., Szell, M., Sobolevsky, S., Strogatz, S. H., & Ratti, C. (2014). Quantifying the benefits of vehicle pooling with shareability networks. *Proceedings of the National Academy of Sciences*, *111*(37), 13290–13294.

Sun, H., Wang, H., & Yang, H. (2023). *Vehicle-based multi-services (VeMuS) for future urban mobility: Concept and research visions* (Available at SSRN 4557634). https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4557634

Vazifeh, M. M., Santi, P., Resta, G., Strogatz, S. H., & Ratti, C. (2018). Addressing the minimum fleet problem in on-demand urban mobility. *Nature*, *557*(7706), 534–538.

Yang, D., Hyland, M. F., & Jayakrishnan, R. (2024). Tackling the crowdsourced shared-trip delivery problem at scale with a novel decomposition heuristic. *Transportation Research Part E: Logistics and Transportation Review*, *188*, 103633.

Acknowledgment: This research is supported by the National Research Foundation (NRF), Prime Minister's Office, Singapore under its Campus for Research Excellence and Technological Enterprise (CREATE) programme. The Mens, Manus, and Machina (M3S) is an interdisciplinary research group (IRG) of the Singapore MIT Alliance for Research and Technology (SMART) centre.