Multi-stage distributionally robust optimization for pre- and post-disaster humanitarian logistics with information constraints

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1 INTRODUCTION

Human suffering and economic losses after a disaster are highly affected by the humanitarian logistics (HL) planning process. It is generally divided into pre-disaster planning and post-disaster operation. The former includes facility location and pre-positioning, while the latter includes inventory-distribution and fleet management. Although disasters are unpredictable, their devastating impacts can be alleviated by preparing transportation capabilities that can provide relief goods to affected populations immediately after a disaster. However, pre-disaster planning cannot be amended in the short term; thus, it requires careful consideration integrated with post-disaster operation amendable to available limited information.

The integrated pre- and post-disaster HL problems have been extensively studied in the past decades (e.g., Rawls & Turnquist, 2010), and various optimization techniques, such as two- or multi-stage stochastic programming, have been proposed (Dönmez *et al.*, 2021). The first and subsequent stage problems optimize planning and operations, respectively. It should be noted that relief demand is non-stationary, significantly impacting HL operations. The impact further spreads to strategic planning to support efficient operations. Therefore, we develop a multi-stage problem that formulates pre-disaster planning and subsequently determines post-disaster operations sequentially and recursively according to observed random variable realizations.

Given the damaged communication infrastructure and inadequate historical data, existing HL multi-stage models lack (1) appropriate information availability and (2) information ambiguity. Most existing HL models implicitly assume the entire damage is observable after a disaster. In

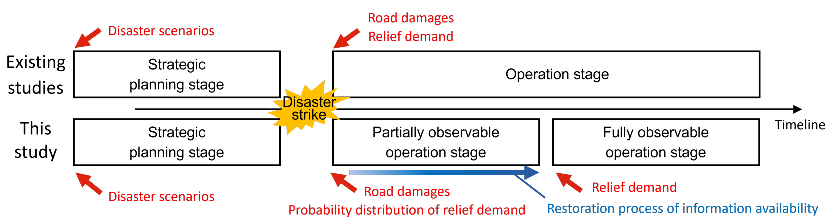


Figure 1 – *Proposed humanitarian logistics planning process*

Table 1 – *Incurred costs, decision variables, and observable random variables at each stage*

Stage	Incurred cost	Decision variable	Observable random variable
Planning	Facility opening cost	Facility location	
	Stockpile holding cost	Pre-positioning	
	Procurement contract cost	Procurement planning	
Partially observable operation	Inventory holding cost	Inventory control	Road damages
	Piecewise-linear deprivation cost*	Relief distribution	Probability distribution of demand
Fully observable operation	Transportation cost	Fleet management	Restoration of communication infra.
	Emergency procurement cost	Emergency procurement	Realizations of cumulative demand

* Not included in the partially observable stage.

contrast to physical damage to infrastructures, accurate information on relief demand may require some time to become clear. Humanitarian organizations, therefore, need to supply relief goods according to probability distributions of relief demand estimated from demographic data and macroscopic damages in early chaotic periods (Kawase & Iryo, 2023). In light of the information availability in practical operations, the HL planning process should be divided into strategic planning, partially observable, and fully observable operation stages, as shown in Figure 1.

Moreover, the ambiguity of probability distributions is inevitable in the HL planning process. It has been revealed that, due to ambiguity, the true probability distribution may produce disappointing performance, and in some cases, relief decisions may be infeasible (Bozorgi-Amiri & Khorsi, 2016). This phenomenon is known as *the Optimizers' Curse*. Distributionally robust optimization (DRO) is a promising solution to alleviate such adverse impacts. To our knowledge, no multi-stage HL models address DRO, even though the dynamic nature of relief demand and restoration of information availability are unique features of HL.

This paper presents a multi-stage DRO model for pre- and post-disaster HL to address information availability and ambiguity. We formulate a DR multi-stage stochastic linear problem (DR-MSSLP), using Wasserstein-based uncertainty sets. The dynamical system is based on the minimum cost flow problem. DRO problems are generally nested min-max problems. Wasserstein uncertainty sets can yield a single-level linear reformulation. Furthermore, leveraging the linearity, a multi-stage Benders decomposition, called Stochastic Dual Dynamic Programming (SDDP), provides lower bounds for the optimal value of DR-MSSLPs (Duque & Morton, 2020).

2 METHODOLOGY

2.1 Problem setting

The proposed model minimizes the system cost incurred within a given planning horizon in a DRO manner. The planning horizon is divided into planning and multi-stage operation stages. The operation stages consist of partially and fully observable stages; when communication infrastructure is restored, the stage transitions from the former to the latter. We assume the restoration process to be uncertain. In the former stage, the probability distribution of relief demand is estimated, while in the latter stage, the past realizations of relief demand are revealed. Table 1 summarizes costs, decision variables, and observable random variables at each stage.

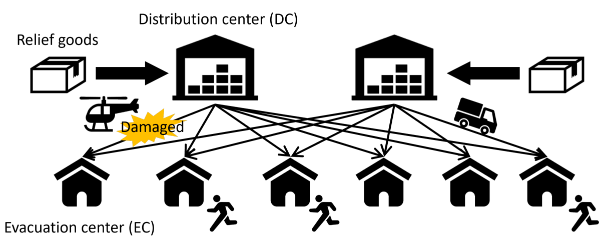


Figure 2 – *An example of humanitarian logistics network*

Relief goods are shipped from distribution centers (DCs) to evacuation centers (ECs) on an HL network, as shown in Figure 2. The geographic relationships between candidate DCs and ECs are known. The facility location and capacities are determined from the candidate DCs. We consider single-packaged relief goods (e.g., water and food). The relief goods flow is explained as follows. DCs supply pre-positioned and additional procured goods to ECs. The additional goods are procured from outside of the affected areas. The relief goods are loaded into trucks and helicopters at DCs, then transported to ECs, and eventually supplied to affected populations. The goods flow is constrained by facility inventory, transshipment, and vehicle capacity.

Trucks and helicopters deliver relief goods to ECs. Trucks are deployed according to vehicle procurement planning, and additional helicopters are procured for delivery to ECs with disrupted road access in the operation stages. Trucks are constrained by road accessibility, facility parking capacity, and road capacity, in contrast to helicopters. Both vehicles have a carrying capacity. The movements of trucks, helicopters, and goods are described as macroscopic flows.

2.2 Formulation

We consider the following dynamic programming (DP) equations for the DR-MSSLP:

$$\min_{\boldsymbol{x}_1 \geq \boldsymbol{0}, \boldsymbol{y} \in \{\boldsymbol{0},\boldsymbol{1}\}} \Big[F_1(\boldsymbol{x}_1, \boldsymbol{y}) + \max_{\mathcal{Q}_2 \in \mathbb{Q}_2(\mathcal{P}_2)} \mathbb{E}_{\mathcal{Q}_2} [V_2(\boldsymbol{x}_1, \boldsymbol{y}, \boldsymbol{\xi}_2)] \Big], \tag{1}$$

$$V_t(\boldsymbol{x}_{t-1}, \boldsymbol{y}, \boldsymbol{\xi}_t) = \min_{\boldsymbol{x}_t \in \mathcal{X}_t(\boldsymbol{x}_{t-1}, \boldsymbol{y}, \boldsymbol{\xi}_t)} \Big[F_t(\boldsymbol{x}_t) + \max_{\mathcal{Q}_{t+1} \in \mathbb{Q}_{t+1}(\mathcal{P}_{t+1})} \mathbb{E}_{\mathcal{Q}_{t+1}} [V_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}, \boldsymbol{\xi}_{t+1})] \Big], \quad \text{where} \;\; V_{T+1}(\cdot) = 0 \;\; \forall t. \tag{2}$$

F_1 and $F_{t=2,...,T}$ denote linear planning and operation costs, respectively. $\mathbb{E}_{\mathcal{Q}_t}$ is the conditional expectation operator with the probability distribution $\mathcal{Q}_t$ at stage t. $\boldsymbol{y}$ indicates facility location and $\boldsymbol{x}_t$ indicates other continuous decision variables at stage t. $\mathcal{X}_t$ is a set of linear constraints given random variables $\boldsymbol{\xi}_t$ and decision variables $(\boldsymbol{x}_{t-1}, \boldsymbol{y})$, including the dynamical equations of inventories and shortages, flow conservation constraints, and capacity constraints. $\mathbb{Q}_t(\mathcal{P}_t)$ is a Wasserstein-based uncertainty set given by nominal distributions $\mathcal{P}_t = (p_t^i | \forall i \in [n_t])$ on prespecified finite support, $\{\boldsymbol{\xi}_t^1, ..., \boldsymbol{\xi}_t^{n_t}\}$, with n_t realizations of random variables.

The dualization of the Wasserstein metric yields a single-level linear reformulation of Eqs. (1)(2). Under the finite support, the Wasserstein-based uncertainty set is given by

$$\mathbb{Q}_{t+1}(\mathcal{P}_{t+1}) = \Big\{ \mathcal{Q}_{t+1} = (q_t^j | \forall j \in [n_{t+1}]) \Big| \max_{\boldsymbol{z}_t \geq \boldsymbol{0}, \boldsymbol{q}_{t+1} \geq \boldsymbol{0}, \sum_j z_t^{ij} = p_{t+1}^i} \textstyle\sum_{ij} d_{t+1}^{ij} z_t^{ij} \leq \alpha \Big\} \;\; \forall t, \tag{3}$$

$$\textstyle\sum_i z_t^{ij} = q_{t+1}^j \quad \forall t, j \in [n_{t+1}], \tag{4}$$

where $d_t^{ij} = ||\xi_t^i - \xi_t^j||_{\delta \in \{1,2,\infty\}}$ and α is the maximum allowable Wasserstein metric. A larger α produces a larger uncertainty set, which represents a risk-averse preference. The inner problem is a maximization problem with Eqs. (3)(4) as linear constraints. Therefore, let θ_t and $\boldsymbol{\omega}_t$ be dual variables of Eqs. (3)(4), and we can reformulate Eq. (2) as follows:

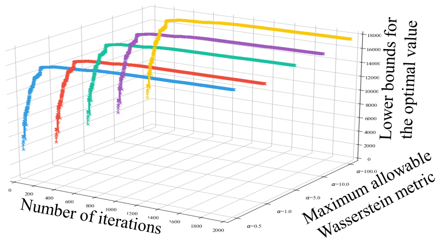
$$V_t(\boldsymbol{x}_{t-1}, \boldsymbol{y}, \boldsymbol{\xi}_t) = \min_{\boldsymbol{x}_t \in \mathcal{X}_t(\boldsymbol{x}_{t-1}, \boldsymbol{y}, \boldsymbol{\xi}_t), \theta_t \geq 0, \boldsymbol{\omega}_t} [F_t(\boldsymbol{x}_t) + \alpha \theta_t + \boldsymbol{p}_{t+1} \boldsymbol{\omega}_t^\top] \;\; \forall t, \tag{5}$$

$$\text{s.t.} \;\; d_{t+1}^{ij} \theta_t + \omega_t^i \geq V_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}, \boldsymbol{\xi}_{t+1}) \;\; \text{and} \;\; V_{T+1}(\cdot) = 0 \;\; \forall t, i \in [n_{t+1}], j \in [n_{t+1}]. \tag{6}$$

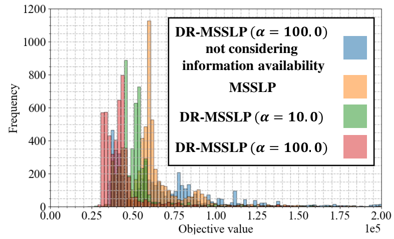
We make the following assumptions for all stages to solve Eqs. (5)(6):

1. Random variables are stagewise independent, i.e., $\boldsymbol{\xi}_{t+1}$ is independent of $[\boldsymbol{\xi}_2, ..., \boldsymbol{\xi}_t]$, and
2. $\mathcal{X}_t(\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_t)$ is almost surely non-empty for every $(\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_t)$.

Under the above assumptions, SDDP can yield lower bounds for the optimal value of Eqs. (5)(6) with guaranteed convergence (Duque & Morton, 2020). It is an iterative algorithm that generates Benders cuts of the solution to DP equations by utilizing the duality of MSSLPs. The assumption of stagewise independence relaxes the curse of dimensionality since it generates



(a) *Convergence processes of lower bounds for the optimal values of DR-MSSLPs*



(b) *Histograms of out-of-sample objective values*

Figure 3 – *Numerical results*

Benders cuts without tracking all scenarios. Our model assumes the observability of stochastic demand depends on past restoration processes, which violates Assumption 1. SDDP can avoid this issue by discretizing the random data process to a Markov chain (Löhndorf & Shapiro, 2019). Assumption 2 can be satisfied by introducing slack variables into the DR-MSSLP.

3 NUMERICAL EXPERIMENTS

Numerical experiments show the verification of the proposed model and the impact of information availability and ambiguity, using the HL network shown in Figure 2. The vehicle capacity was based on trucks and helicopters used in the 2024 Noto Peninsula Earthquake. Cost coefficients and facility inventory capacities were according to the previous works (e.g., Rawls & Turnquist, 2010). At each stage, the nominal distribution of relief demand was given by a uniform distribution, and the nominal probability of communication infrastructure restoration was given by 0.7. 100 realizations were sampled at each stage to define finite supports. Relief demand was set to arise until t=10. This means the total number of scenarios is approximately 100^{10}. We should note that lower bounds for the optimal value are formed without tracking all scenarios.

Figure 3a illustrates the convergence process of lower bounds in the case of some Wasserstein uncertainty sets. Figure 3b shows the performance of policies constructed by DR-MSSLPs for 5000 out-of-samples. Figure 3b demonstrates that consideration of information availability and ambiguity in the HL model contributes significantly to improving its out-of-sample performance. It should be noted that the only DRO approach might not be sufficient to address *the Optimizers' Curse* caused by inappropriate considerations of information availability after a disaster.

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