Robust Outbound Load Planning with Volume Splitting for Parcel Carriers

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1 INTRODUCTION

Parcel carriers like UPS and FedEx must manage rapid growth in package demand due largely to e-commerce; analysts project that today's $3.3 trillion e-commerce market could grow to $5.4 trillion annually by 2026 (Morgan Stanley, 2022). Parcel carriers operate highly-effective first-mile and last-mile operations that interface directly with customers for induction (pickup) of packages and final delivery. In this paper, however, we focus on the ground-transportation middle-mile consolidation networks of such carriers. The middle-mile (or linehaul) network uses trucking and intermodal rail transportation to move packages between origin and destination hubs through intermediate sorting hubs. Packages are directed through the middle-mile network according to a *flow plan* and moved by trailers and containers determined by a *load plan*.

Flow plans typically aggregate package volume by final terminal destination and due date (or service class) and specify consistent rules (often differentiated by day-of-week or even specific date) for routing flow between hubs such that packages arrive to destinations on time. Hubs operate multiple discrete sorting periods (or sorts) each day, and load plans specify the number and types of trailers or containers to build outbound from each hub during each sort. While flow plans are adjusted infrequently, load plans are often adjusted weekly.

Flow plan optimization is a planning problem for package carriers that is to select a *primary flow path* for all package volumes to be handled by the network. If we denote the aggregated package volume for a final destination and due date as a *commodity*, the primary flow paths can be specified by selecting a unique outbound next load (hub,sort) for each commodity at each sorting hub during each sort. When a carrier operates a large network with many terminals, it is often true that some commodities may be routed onto *alternate flow paths* when they reach some hub during some sort. These alternate paths specify different next load lanes than the primary, but commodity volume remains projected to arrive at the final destination on time.

Jointly optimization of primary and alternate flow path decisions for all commodities across a network seeks to minimize the expected load transportation cost and package volume sorting costs. For large-scale networks, traditional time-space service network design optimization formulations are intractable for this optimization. This paper considers a simpler subproblem focused on specifying a low-cost load plan outbound from a single hub for a series of consecutive sorting periods given pre-specified primary and alternate flow paths for the volumes of each commodity arriving to the hub. Unlike most prior work on this problem, here we take advantage the opportunity to *split* commodity volumes across primary and alternate paths. Leveraging feasible alternate paths can provide major reductions in required network capacity in the load plan.

This outbound load planning problem with volume splitting is solved a few days before the day of operations, and therefore we develop and solve in this paper a robust variant. Extending the work in Ojha *et al.* (2023), we propose the robust outbound load planning problem (ROLPP) and model it as a two-stage robust optimization model with relatively complete recourse and make the following contributions:

- We solve the ROLPP using a master-subproblem framework embedded in an exact column-and-constraint (CCG) algorithm. We reformulate the *max-min* bilinear subproblem as a mixed-integer linear (MILP) program with *BigM* constraints.
- We propose heuristic scenario generation ideas that leverage the structure of the problem; these scenarios, when added to the master problem, provide tight lower bounds to the optimal ROLPP objective without having to solve the computationally intensive MILP formulation of the subproblem.
- We integrate the heuristic scenario generation step with a mountain climbing heuristic to efficiently generate local optimal solutions to the subproblem in the CCG algorithm. The local optimal solutions can be used as effective warm starts for the MILP reformulation of the subproblem; it is sometimes difficult for the commercial solver to produce a feasible solution to the subproblem within 600 seconds. On average, the local optimal solutions are within $11\% - 17\%$ of the best bound obtained at the root node of the *branch-and-bound* tree when solving the subproblem using a commercial solver.
- We conduct an extensive computational study on real-life data for a medium-sized terminal of a large parcel carrier in the US. We show that having both primary and alternate routing options for commodities is more effective in managing the cost of recourse actions (when a demand scenario from a given uncertainty set is revealed) than when commodities only have a primary routing option.

2 MODELING AND SOLUTION APPROACH

2.1 Robust OLPP Formulation

Consider hub o with multiple sorting periods on a given day. Define A as the set of outbound load lanes from o, where each $a \in A$ departs o during some sort and is destined for some next hub during some subsequent sort. Let K be the commodities arriving to o for outbound loading, each arriving during some sort with volume q^k. Let $A^k \subseteq A$ be the set including the primary and all alternate outbound loads for commodity k.

Volume from commodity k can be assigned to $a \in A$ at per-unit cost d_a^k; the primary lane is cheapest. For ease of notation, suppose that each lane operates only one trailer type with capacity Q. Dispatching a single trailer on lane a incurs cost c_a. Variables y_a denote the integer number of trailers to dispatch on each arc a while continuous variables x_a^k allocate the volume of commodity k to arcs. Given commodity volumes, the following formulation minimizes the cost of the load plan:

$$
\begin{aligned}
\underset{x,y}{\text{Minimize}} \quad & \sum_{a \in A} c_a y_a + \sum_{k \in K} \sum_{a \in A^k} d_a^k x_a^k && && \text{(1a)}\\
\text{subject to} \quad & \sum_{a \in A^k} x_a^k = q^k, && \forall k \in K, && \text{(1b)}\\
& \sum_{k \in K : a \in A^k} x_a^k \leq Q\, y_a, && \forall a \in A, && \text{(1c)}\\
& x_a^k \geq 0 && \forall k \in K, a \in A^k, && \text{(1d)}\\
& y_a \in Z_{\geq 0} && \forall a \in A. && \text{(1e)}
\end{aligned}
$$

The objective function minimizes the costs of moving loads plus penalties to favor the primary lane or preferred alternates. The constraints allocate all commodity volume to compatible lanes and ensure that enough loads are dispatched on the lanes to containerize all of the volume.

Now suppose that commodity volumes lie within an uncertainty set $\mathcal{Q}$:

$$\mathcal{Q} := \{\hat{\mathbf{q}} : q^k \leq \hat{q}^k \leq (1 + u^k)q^k \;\forall\, k \in K, \sum_{k\in K}(\hat{q}^k - q^k) \leq B\} \tag{2}$$

where commodity volumes may increase from their nominal values by a maximum inflation of u^k but a budget B controls the total inflation across commodities.

A robust variant of the problem is now given by (3):

$$\underset{\mathbf{y}\in Z_{\geq 0}}{\text{Min}} \quad c^T\mathbf{y} + \underset{\hat{\mathbf{q}}\in\mathcal{Q}}{\text{Max}} \quad \underset{(\mathbf{x},\mathbf{z})\in F(\mathbf{y},\hat{\mathbf{q}})}{\text{Min}} (w^T\mathbf{z} + d^T\mathbf{x}) \tag{3}$$

where $\mathbf{y}$ remain the planned loads on each lane, but the allocation $\mathbf{x}$ of commodities to lanes is now a recourse decision after the uncertainty demand is revealed and $\mathbf{z}$ captures lanes that exceed capacity and penalizes them. Feasible solutions to the recourse problem lie in the feasible space $F(\mathbf{y}, \hat{\mathbf{q}})$ defined by the following constraints:

$$\sum_{a\in A^k} x_a^k = \hat{q}^k, \quad \forall\, k \in K \tag{4a}$$

$$\sum_{k:a\in A^k} x_a^k \leq Q_a(y_a + z_a), \quad \forall\, a \in A \tag{4b}$$

$$x_a^k \geq 0, \quad \forall\, k \in K, a \in A^k \tag{4c}$$

$$z_a \geq 0, \quad \forall\, a \in A \tag{4d}$$

2.2 Solving the Robust OLPP

To solve the robust model, we propose a column-and-constraint generating procedure (CCG) following the approach of Zeng & Zhao (2013). We first consider the bi-level optimization problem faced by the adversary given by:

$$S(\mathbf{y}) := \underset{\hat{\mathbf{q}}\in\mathcal{Q}}{\text{Max}} \quad \underset{(\mathbf{x},\mathbf{z})\in F(\mathbf{y},\hat{\mathbf{q}})}{\text{Min}} (w^T\mathbf{z} + d^T\mathbf{x}) \tag{5}$$

and replace it with a single bilinear optimization problem by replacing the inner minimization problem with its linear programming dual. This bilinear formulation can be further simplified by including its KKT conditions as non-linear constraints which can then be linearized by formulating the problem as a mixed-integer linear program. Given any vector of planned loads $\mathbf{y}$, this adversary MIP can then be solved to identify an extreme point scenario in the uncertainty set $\mathcal{Q}$ and the resulting worst-case recourse cost.

To create the full solution approach, we now note that the uncertainty set $\mathcal{Q}$ has a finite set of extreme points. Furthermore, we can show that we need only optimize over the extreme points of $\mathcal{Q}$ to find an optimal robust feasible solution. This motivates the following CCG algorithm. In each iteration, we solve a master problem with a subset of extreme points to minimize the cost of the integer load vector $\mathbf{y}$ and its associated recourse cost given a subset of all extreme points of the uncertainty set. The master problem always provides a lower bound. Then, the bilinear problem is solved with the capacity vector $\mathbf{y}$ which provides both an upper bound and possibly a new extreme point to add to the master problem. We repeat until convergence.

A good feature of this algorithm is that we can use smart heuristics to speed up solution times. First, we can generate reasonable sets of uncertainty set extreme points to include in the initial master problem. Second, we can also deploy smart approaches to solve the subproblem to find a new extreme point in the uncertainty set to improve the lower bound.

3 RESULTS and DISCUSSION

We conduct experiments using data provided by our research partner for a medium-sized hub terminal for one day. There are 4,425 commodities with a total cubic volume of $\sum_{k \in K} q^k = 140,000$ in the nominal instance. The terminal creates outbound loads on 64 lanes A. Only short 23-foot pup trailers are used. About one-third of commodities have only a primary flow load lane, while two-thirds have two or more.

We solve the robust formulations, varying the amount of possible commodity volume increase using two parameters Δ and β, where Δ controls the maximum percentage increase of any individual commodity volume and β controls the maximum percentage increase of the total commodity volume (given Δ). When $\Delta = \beta = 0$, we do not plan for robustness but we note that the model can still pay penalties to exceed demand on some lanes.

Instance	Δ	β	$c^T\mathbf{y}^*$	η^*	$w^T\mathbf{z}^*$	$d^T\mathbf{x}^*$	LB	UB	Gap	MPT(s)	SubpT(s)	TotalT(s)	#Iter
1	0.0	0.0	8,179	1,541.18	925.27	615.91	9,720.18	9,720.18	0.0	0.37	24.84	25.2	2
2	0.2	0.2	9,269	1,557.58	913.89	643.68	10,826.58	10,827.96	0.01	6.84	1,298.4	1,305.23	5
3	0.2	0.4	9,493	1,546.58	815.61	730.97	11,039.58	11,049.46	0.09	8.14	1,230.16	1,238.3	5
4	0.2	0.6	9,607	1,564.2	829.4	734.81	11,171.2	11,173.37	0.02	12.16	1,464.61	1,476.77	5
5	0.2	0.8	9,676	1,587.39	831.36	756.03	11,263.39	11,263.47	0.0	1.75	979.66	981.41	3
6	0.2	1.0	9,699	1,571.2	815.61	755.58	11,270.2	11,271.56	0.01	0.34	317.86	318.2	2

Table 1 – *Statistics for the CCG algorithm when commodities have primary and alternate routing options*

Table 1 summarizes the results when commodities are allowed to use alternate routing options and up to a 20% increase in the volume of any individual commodity may be oberved. Robust solutions are found in reasonable running times (seconds) and the algorithm converges to small optimality gaps. It is important to note that to provide robust protection, planned load plan costs increase by 13% or more.

Instance	Δ	β	$c^T\mathbf{y}^*$	η^*	$w^T\mathbf{z}^*$	$d^T\mathbf{x}^*$	LB	UB	Gap	MPT(s)	SubpT(s)	TotalT(s)	#Iter
1	0.0	0.0	28,911	3,571.98	3,571.98	0	32,482.98	32,482.98	0.00	0.13	0.06	0.18	2
2	0.2	0.2	31,776	5,161.46	5,161.46	0	36,937.46	36,948.06	0.03	1.76	0.72	2.48	12
3	0.2	0.4	32,787	4,718.8	4,718.8	0	37,505.8	37,539.0	0.09	1.84	0.95	2.79	14
4	0.2	0.6	33,035	4,539.46	4,539.46	0	37,574.46	37,574.46	0.00	0.26	0.29	0.54	6
5	0.2	0.8	33,088	4,467.11	4,467.11	0	37,555.11	37,555.11	0.00	0.12	0.24	0.36	3
6	0.2	1.0	33,088	4,467.11	4,467.11	0	37,555.11	37,555.11	0.00	0.03	0.07	0.09	2

Table 2 – *Statistics for the CCG algorithm when commodities have only one primary routing option*

It is interesting to see the value of alternate flow paths for routing commodity volume outbound from a terminal, especially when robust plans are desired. Table 2 summarizes the results when commodities must be routed on the primary path. In this case, many lanes are stuck with small fractional trailerloads that must be dispatched long distances leading to significantly higher costs. Not surprisingly, however, the relative costs of robust plans are lower in the case reflecting the fact that spare capacity already exists since no alternate consolidation is allowed.

References

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