Drone Delivery Network Design with Uncertainties

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1 INTRODUCTION

Unmanned aerial vehicles (UAVs), also called drones, are gaining popularity as an alternative delivery mode due to their faster delivery speed and reduced labor costs. Several companies, especially e-commerce giants, are conducting pilot projects that use drones to deliver fast food and groceries. In 2021, for example, Walmart partnered with Zipline in the United States to provide delivery services for areas near Walmart stores in Arkansas. In China, Meituan drone delivery services have been launched in Shenzhen and have conducted trial food delivery that cover more than 8,000 households.

Drones are expected to play an important role in future urban logistics and transportation. It is thus increasingly essential that drone delivery network infrastructures be constructed to enable drones to load and unload parcels, take off and land, charge, and park. How to design a drone delivery network has attracted increasing attention in recent years, which is studied as the drone-based delivery network design problem (DDNDP). Pinto & Lagorio (2022), Enayati et al. (2023), and Liu (2023) have investigated and designed strategies for the practical deployment of drone facilities, such as charging stations and docking hubs, to address difficulties caused by limited drone capacity and flight range. However, research gaps remain and present three main challenges: (i) managing complex operations that involve the pickup and delivery of parcels, the critical limitation of battery charging for drones, and the time-window requirements for deliveries; (ii) navigating the flight arc (e.g., air route) accessibility uncertainty in urban areas due to factors such as weather conditions and air traffic control and regulations; and (iii) addressing the urban delivery demand uncertainty with limited precedent. Specifically, in urban scenarios, high population density can cause sudden surges in demand, which are difficult to predict with precision. Moreover, unlike traditional modes, the data of flight arc accessibility for the drone delivery is limited. Integrating demand and arc accessibility uncertainties into network design is thus essential for fostering system flexibility and robustness.

This paper studies a drone delivery network problem in an urban low-altitude scenario that involves tactical decisions on hub locations, capacities, and the establishment of flight arcs. Our objective is to minimize the investment cost of network infrastructures and the operational costs of drone delivery while accounting for both soft and hard time windows for parcels and the capacity of drone parking and charging facilities. To address the uncertainty in demand and air route conditions, we propose a two-stage distributionally robust optimization approach, which employs a distribution separation procedure and split cuts in a combined Benders decomposition and column-and-constraint generation algorithm. The effectiveness and robustness of the proposed method are demonstrated through extensive experiments.

2 PROBLEM STATEMENT

We consider a drone delivery system that operates a fleet of drones to cater to uncertain flight arc accessibility and demands. We define the set of demands as K, indexed by k. Each demand comprises a pick-up node s_k, a drop-off node f_k, a time window $[W_k, U_k]$, and a delivery amount D_k^{ω}. Notably, the amount D_k^{ω} is subject to uncertainty, with ω ranging over a finite sample space Ω. We establish a directed graph $G = (N, A)$, where N and A are the node set and arc (i.e., route) set, respectively. Node set N includes the set of candidate hubs H and the set of customer nodes J, which include pick-up and drop-off nodes. The condition of arc (i, j) under each uncertain scenario γ is defined as C_{ij}^{γ}, which denotes the accessibility of the arc. For each hub, we define a fixed cost F^H, a unit capacity cost F^C. We also define a fixed cost F_{ij} for each arc.

We presume that the drone delivery system operates under the following assumptions:

1. A drone can serve at most one customer per trip.
2. The drone's battery is replaced with a fully charged one when it returns to the hub.
3. The capacity of a drone is limited to one unit.
4. The hub from which a drone departs and the one it returns to can be different.
5. The demand can be split and served by multiple drones or by one drone for multiple trips.

Assumptions 1–3 stem from the recognized constraints on drone flight endurance, which are commonly used in both practical applications and literature (Murray & Chu, 2015); assumptions 4–5 allow for the sharing of drones between hubs and the collaborative fulfillment of demand by multiple drones, which would enhance operational flexibility in reality.

We aim to design a drone delivery network that minimizes overall investment and operational costs. Our problem can be divided into two stages. The first-stage model determines the drone network, which involves three types of decision variables: (i) z_i on whether hub i is active or not; (ii) y_{ij} on whether arc (i, j) is active or not; and (iii) q_i on the capacity of hub i. The second-stage model is for routing drones to fulfill uncertain demands under operational constraints while considering the arc accessibility and capacity uncertainty. The decision variables include the number of drones that pass through each arc at each time period.

2.1 Mathematical Model

The first-stage model determines the drone network. The objective function

$$\min_{\boldsymbol{y},\boldsymbol{z},\boldsymbol{q}} \sum_{i\in H} z_i F^H + \sum_{(i,j)\in A} y_{ij} F_{ij} + \sum_{i\in H} q_i F^C + \mathbb{E}_{\widehat{\mathbb{P}}}[Q(\boldsymbol{y},\boldsymbol{z},\boldsymbol{q},\omega,\gamma)]$$

minimizes the total cost, which includes the investment cost of the hubs and air routes (arcs) and the expected operational cost from the second stage, where $\widehat{\mathbb{P}}$ is the distribution of the uncertainties.

The second-stage model is for routing drones, with the objective $Q(\boldsymbol{y},\boldsymbol{z},\boldsymbol{q},\omega,\gamma)$ minimizing expected general operational costs while accounting for operational constraints, demand fulfillment constraints, and hub and arc capacity.

3 Methodology

3.1 Stochastic programming problem

In practice, we generally do not know the true distribution $\widehat{\mathbb{P}}$ that characterizes demand and arc accessibility and capacity. Some historical data on ground delivery can provide reference into the uncertain air delivery demands. As for the arc conditions, we can only make assumptions by considering historical environmental data and possible regulations. By substituting the term $\mathbb{E}_{\widehat{\mathbb{P}}}[\cdot]$ in the first-stage model with $\mathbb{E}_{\mathbb{P}^*}[\cdot]$, we first build a stochastic programming model, where $\mathbb{P}^*$ denotes the stochastic distribution constructed using historical data and assumptions. For convenience, we define all variables in the first-stage model as $\boldsymbol{x}$ and those in the second-stage model as $\boldsymbol{a}$. The stochastic programming model can be thereby written as: $\min_{\boldsymbol{x}} f(\boldsymbol{x}) + \mathbb{E}_{\mathbb{P}^*}[\mathcal{Q}(\boldsymbol{x}, \omega, \gamma)]$. The model can then be solved by using the sample average approximation (SAA) method.

3.2 Distributionally robust optimization

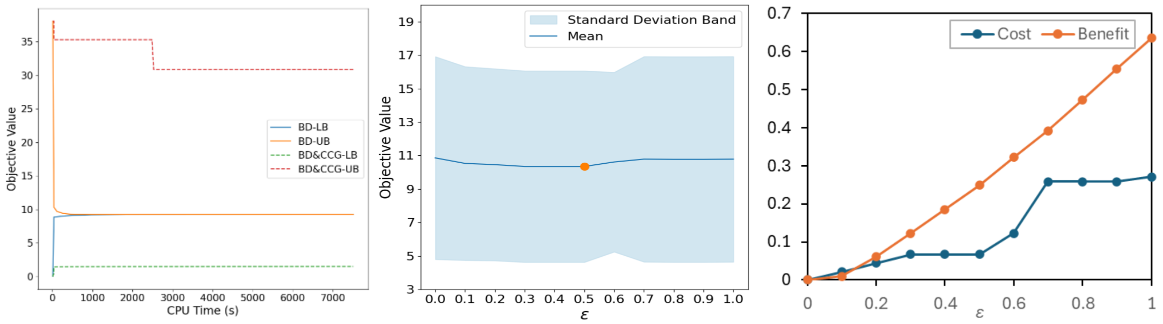
In reality, the data for air delivery are scarce. On the demand side, we only have some ground delivery data for reference. On the supply side, the flight arc accessibility and capacity, which is affected by air traffic control and weather conditions, is rare, if not completely unknown. Therefore, the distribution generated by historical data and assumptions may be far from the true distribution (Wang et al., 2020). To deal with this issue, distributionally robust optimization (DRO) mitigates overfitting to historical datasets and assumptions by incorporating an ambiguity set that encompasses all possible distributions.

By setting the function $\mathbb{E}_{\widehat{\mathbb{P}}}[\cdot]$ as $\max_{\mathbb{P} \in \mathfrak{B} \times \mathcal{F}} \mathbb{E}_{\mathbb{P}}[\cdot]$, the DRO model is thereby written as $\min_{\boldsymbol{x}} \left\{ \boldsymbol{f}^{\top}\boldsymbol{x} + \max_{\mathbb{P} \in \mathfrak{B} \times \mathcal{F}} \mathbb{E}_{\mathbb{P}}[\mathcal{Q}_{\omega,\gamma}(\boldsymbol{x})] \,|A\boldsymbol{x} \geq b, \boldsymbol{x} \in \mathbb{Z} \right\}$, where $\mathfrak{B}$ and $\mathcal{F}$ are the Wasserstein distance-based ambiguity set for demand and arc accessibility uncertainty, respectively. Due to the space limitation, we express the subproblem in a standard form here: $\mathcal{Q}_{\omega,\gamma}(\boldsymbol{x}) = \min_{\boldsymbol{a}} \left\{ \boldsymbol{g}_{\omega,\gamma}^{\top} \boldsymbol{a}_{\omega,\gamma} | W_{\omega,\gamma} \boldsymbol{a}_{\omega,\gamma} \geq \boldsymbol{r}_{\omega,\gamma} - T_{\omega,\gamma} \boldsymbol{x}, \boldsymbol{a}_{\omega,\gamma} \in \mathbb{Z} \right\}$.

To solve this problem, we first relax the two-stage model into a linear programming problem, then add it to the first-stage model in the form of a Benders cut. The form of the cut is expressed as $\theta \geq \sum_{\omega \in \Omega} \sum_{\gamma \in \Gamma} v_{\omega} o_{\gamma} \left\{ \pi_{\omega,\gamma}^{*}(\boldsymbol{x})^{\top} (\boldsymbol{r}_{\omega,\gamma} - T_{\omega,\gamma}\boldsymbol{x}) \right\}$, where θ represents the worst-case objective value of the second stage. Here $\pi_{\omega,\gamma}^{*}(\boldsymbol{x})$ is the optimal dual multipliers for the constraints of the second-stage model and v_{ω} and o_{γ} are the worst-case probabilities obtained by the distribution separation model $\min_{\boldsymbol{v}} \left\{ \sum_{\omega \in \Omega} \sum_{\gamma \in \Gamma} v_{\omega} o_{\gamma} \, \mathcal{Q}(\boldsymbol{x}, \omega) | \, \boldsymbol{v} \in \mathfrak{B} \right\}$. We design an algorithm framework to iteratively add Benders cuts. To further improve the algorithm's performance, we also introduce Column-and-Constraint Generation (CCG) technology, which would tighten the model by adding several variables and constraints of the second-stage problem into the first-stage model.

4 NUMERICAL RESULTS

We conducted extensive numerical experiments using a dataset comprising 8 days of daily operational data for food delivery from Meituan in Beijing, with candidate hubs generated by K-means clustering. The flight arc accessibility data was generated from a uniform distribution.

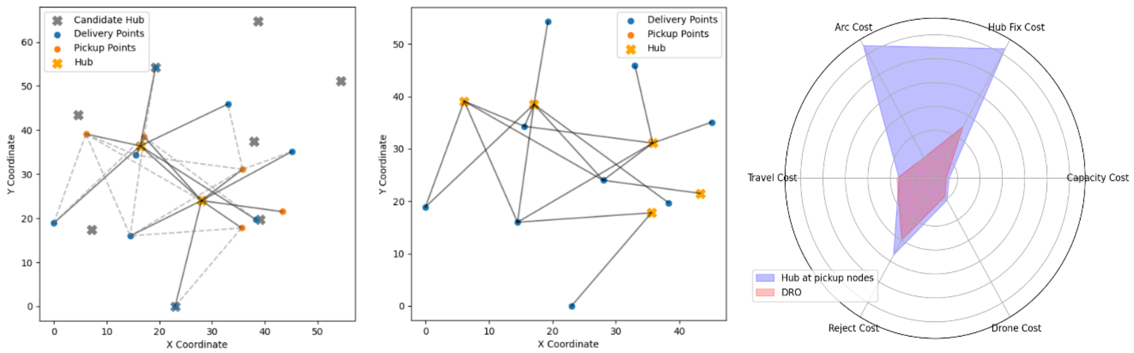


(a) Convergence curves (b) Performance under various ε (c) Cost-benefit test

Figure 1 – *Test performance of DRO*

We first test the performance of DRO and our proposed method. Figure 1(a) shows that the DRO method converges in only 5 iterations using both Benders and CCG methods, achieving an optimal objective value of 8.915. In contrast, when only Benders cuts are used, the gap remains at 94.4% after 7200 seconds, with an objective value of 28.480. This highlights that CCG significantly accelerates convergence by providing a better estimation of the second-stage objective function. Figure 1(b) examines the robustness of the DRO solution by varying the ambiguity set deviation ε. The results indicate that the best mean value is achieved at $\varepsilon = 0.5$. Furthermore, the cost-benefit analysis shown in Figure 1(c) reveals that as uncertainty increases, the benefits of DRO rise significantly, while the cost to maintain robustness increases slightly.

Figures 2 compare the delivery network generated by DRO and by placing hubs at pickup nodes. The radar chart in Figure 2(c) shows that placing hubs at pickup nodes leads to high investment costs without reducing operational costs. Although travel costs slightly decrease, the demand rejection and drone usage costs increase due to inefficient drone sharing and scheduling.



(a) Network of DRO (b) Basic network with hubs at pickup nodes (c) Radar chart

Figure 2 – *Case study of different delivery networks*

5 CONCLUSION

This paper study the design of a drone delivery network with uncertain demand and flight arc accessibility. The proposed two-stage distributionally robust optimization approach has proven to be effective in minimizing investment and operational costs while accommodating the complexities of customer time windows and drone capacity constraints. The extensive experiments conducted have validated the robustness and effectiveness of our method. In the full version of the paper, we will provide a comprehensive introduction to the complete model, detailed descriptions of the algorithm and acceleration techniques, a complete set of experiments, and an in-depth analysis of the solutions generated by our models. Additionally, we will offer several management insights aimed at optimizing decision-making in real-world applications.

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