Pricing carsharing services under decision-dependent demand uncertainty: A two-stage stochastic programming approach

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1 INTRODUCTION

Carsharing pricing decisions have attracted significant attention in the research literature. They have been identified as a promising instrument to resolve fleet imbalances Illgen & Höck (2019), and improve profits and service rates. Among other things, prices are commonly optimized under deterministic demand assumptions Jorge *et al.* (2015). Few studies have taken the demand stochasticity into consideration when optimizing pricing decisions; examples include Lu *et al.* (2018), Huang *et al.* (2021) and Pantuso (2022). Still, these studies address carsharing demand as an *exogenous* random variable described by a given probability distribution (e.g., Poisson distribution). The existing work does not account for the influence of carsharing services on customers' adoption probabilities, which however makes the random demand dependent on prices.

In this study, **the research goal** is to address the carsharing pricing and relocation problem with *endogenous* demand uncertainty. Particularly, pricing decisions cause shifts in the probability distribution of random demand. This type of endogenous uncertainty corresponds to Type-I according to the classification in Goel & Grossmann (2004). For a one-way carsharing system captured by graph $\mathcal{G} := (\mathcal{I}, \mathcal{A})$ with $\mathcal{I}$ representing the carsharing stations and $\mathcal{A}$ containing directed arcs between any pair of stations, we have random demand $\xi : \Omega \rightarrow \mathbb{Z}^{|\mathcal{A}|}$ on each arc $(i, j) \in \mathcal{A}$. The dependency of random demand ξ on pricing decision denoted as x makes the distribution function conditional on x, which is denoted as P_x. Given price-dependent random demand, the decision maker (i.e., carsharing system operator) seeks to optimally set carsharing service prices at different locations, and operate necessary relocation for maximizing profits for a target time period.

The contribution of this research is fourfold.

- We propose a carsharing pricing and relocation problem with decision-dependent demand uncertainty.
- We develop a *a two-stage mixed-integer non-linear stochastic program* to address the pricing and relocation problem, where the random demand ξ follows the price-dependent probability distribution P_x.
- A tailored L-shaped method which accounts for decision-dependent distributions is developed for retrieving exact solutions of the stochastic program. The algorithm extends the scalability of solving this type of problem, comparing to off-the-shelf solvers (e.g., Gurobi).

- Extensive numerical studies have been executed on instances of real-world scale by accounting for empirical price-dependent carsharing demand distributions.

2 Method

2.1 Mathematical Modeling

Let $\mathcal{Z}$ be the set of zones where differentiated pick-up fees can be applied to. The pick-up fee is assumed to be chosen from a set of discrete pricing levels denoted as $\mathcal{L}$. Binary variable x_{zl} takes value 1 if pricing level l is offered at zone z, 0 otherwise. Given a solution $x := (x_{zl})_{z\in\mathcal{Z},l\in\mathcal{L}}$, uncertain demand realizes according to probability distribution P_x. Relocation decisions are assumed to be made simultaneously with the pricing decisions. Let $\mathcal{V}$ be the set of shared vehicles. Binary variable s_{vi} takes value 1 if vehicle v is made available at station i, 0 otherwise. Each relocation operation s_{vi} requires a fixed cost C_{vi}. The general two-stage stochastic program is developed as follows.

$$\max\Big\{-\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} C_{vi}s_{vi} + \mathbb{E}_{P_x}\big[Q(x,s,\xi)\big]\,\big|(x,s)\in\mathcal{X}\subseteq\{0,1\}^{|\mathcal{Z}|\times|\mathcal{L}|}\times\{0,1\}^{|\mathcal{V}|\times|\mathcal{I}|}\Big\} \tag{1}$$

The objective is to maximize expected revenue after subtracting the required costs for relocation. In problem (1), $\mathcal{X}$ represents the set of feasible pricing and relocation decisions. Each recourse problem $Q(x,s,\xi)$ under realization ξ represents the revenue generated by optimally allocating rental cars to materialized demand. In our specification of the recourse problem, it can be formulated as a MILP problem.

Given a first-stage decision x, we define conditional distributions P_x based on a probability space $(\Omega,\mathcal{F},\mu_x)$. Rental demand is defined as a random variable ξ with values in $\mathbb{Z}^{|\mathcal{A}|}$, that is, each ξ_{ij} represents the total rental demand on corresponding arc $(i,j)\in\mathcal{A}$. The probability space $(\Omega,\mathcal{F},\mu_x)$, as well as the way of assigning probabilities to realizations of ξ according to μ_x are introduced in the following.

Figure 1 illustrates the probability space $(\Omega,\mathcal{F},\mu_x)$, where we let random event ω be a vector of 0/1 elements having length equal to the number of customers in the system and representing the withdrawal/occurrence of customers. The set $\mathcal{F}$ of events becomes the power set of Ω. The probability measure μ_x depends on x as indicated by the subscript. Given this setup, we define the random variable ξ as $\xi(\omega) := (\xi_{ij}(\omega))_{(i,j)\in\mathcal{A}}$. Let $i(k)$ and $j(k)$ denote the origin and destination stations of customer k, each $\xi_{ij}(\omega)$ is simply given as

$$\xi_{ij}(\omega) = \sum_{k|i(k)=i,j(k)=j}\omega_k, \quad \forall(i,j)\in\mathcal{A}$$

Using a one-dimension case with 6 customers shown in Figure 1 as an example, the random outcome ω represents the case that only the second and fourth customers choose carsharing, while the others leave the system. This corresponds to the random demand realization $\xi(\omega)=2$.

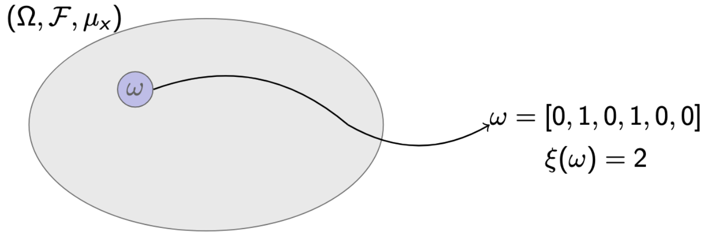

Figure 1 – *Illustration on probability space $(\Omega,\mathcal{F},\mu_x)$ of possible carsharing choice outcomes.*

The distribution function of ξ on $\mathbb{Z}^{|\mathcal{A}|}$ is obtained as follows. For each potential customer k we define the probability of using carsharing given the price set by decisions x as $P(\omega_k = 1|x)$. This probability can be modeled, e.g., using discrete choice models. Therefore, the probability measure μ_x can be defined as

$$\mu_x(\omega) = \prod_{k|\omega_k=1} P(\omega_k = 1|x) \prod_{k|\omega_k=0} (1 - P(\omega_k = 1|x) \tag{2}$$

Finally, the distribution function of ξ given x can be defined as

$$P_x(\xi = N) = \mu_x\left(\{\omega \in \Omega | \xi(\omega) = N\}\right) \tag{3}$$

where N is a nonnegative integer vector.

2.2 Tailored L-shaped method

Problem (1) is solved by a tailored L-shaped method. To make problem (4) tractable, we reformulate this non-linear stochastic program into a problem with finite many probability distribution. Especially for handling the non-linearity contained in function P_x, we partition feasible set $\mathcal{X}$ into a collection of disjoint subsets $(\mathcal{X}_d)_{d\in\mathcal{D}}$, where each subset indicates a distinguished probability distribution P_d. Let $\mathbb{1}_d : \mathcal{X} \to \{0, 1\}$ be the characteristic function of $\mathcal{X}_d$. By incorporating decision-specific distributions, problem (1) is equivalently reformulated as follows:

$$\max_{x,s} \Big\{ -\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} C_{vi}s_{vi} + \sum_{d\in\mathcal{D}} \mathbb{1}_d(x)\mathbb{E}_{P_d}\big[Q(x, s, \xi)\big] \big| (x, s) \in \mathcal{X} \Big\} \tag{4}$$

We solve (4) with one extensive version of the L-shaped method which includes the distribution-specific L-shaped cuts. The method operates by iteratively solving the *Relaxed Master Problem* (RMP) and adding distribution-specific L-shaped cuts at the identified distribution once the optimality condition is violated.

$$\max_{x,s,\phi} \big\{ -\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} C_{vi}s_{vi} + \phi | \phi \geq 0, (x, s) \in \mathcal{X} \big\} \tag{RMP}$$

For accelerating the solving process of the proposed L-shaped algorithm, we delve into strategies of rapid computation of subproblems and valid inequalities. Specifically, exact algorithms are developed for solving the integer recourse problems $Q(x, s, \xi)$. We provide analytical dual solutions of problem $Q(x, s, \xi)$, which facilitate the generation of non-trivial continuous cuts.

3 Results

We generate 9 groups of synthetic instances representing real-world carsharing systems with small, medium and large service regions, each of which is pre-divided into 3, 4, or 5 pricing zones. The service region is represented by a rectangular area composed of $1KM \times 1KM$ small units, the center of which can be viewed as the locations of stations either in station-based or free-floating carsharing systems. We create the small, medium and large service regions with grids of 5×3, 6×4, and 7×5 stations, respectively. Under each instance group, a total number of 36 instances are created with 9 different combinations of vehicle and customer volumes, and 4 different numbers of scenarios, i.e., 5, 20, 50, and 100 per distribution. We assume that the discrete pricing levels range from 0 to 4 Euro with a separation of 1 Euro. The number of distributions (i.e., $|\mathcal{D}|$) is determined by raising the number of pricing levels $|\mathcal{L}|$ to the power of the number of zones $|\mathcal{Z}|$, calculated as $|\mathcal{L}|^{|\mathcal{Z}|}$. In our experiments, we have 125, 625, and 3125 distributions in total for instances with 3, 4, and 5 zones, respectively.

The computational performance of the proposed distribution-specific L-shaped method (D-LS in the following) is compared to that of commercial solver (here, Gurobi 11.0.1) which solves the equivalent deterministic MILP formulation of problem (4) given a 1-hour time limit. For small instances with 15 stations and 5 scenarios per distribution that can be solved to optimality with 10^{-4} tolerance by the solver, we observe that the average solution time is 843.54 seconds, while D-LS solves the corresponding instances with an average time of 14.05 seconds. As the curse of dimensionality intensifies with increasing problem scale, the problem becomes intractable for solver when the number of scenarios exceeds the order of 10^4. This occurs earliest in instances with 4 zones and 20 scenarios per distribution, which result in a total of 1.25×10^4 (calculated by $5^4 \times 20$) scenarios.

For all tested instances, compared to the solver, the D-LS algorithm increases the average percentage of solved instances from 23.77% to 70.37%, as shown in Table 1. This percentage further rises to 78.70% for D-LS when the optimality gap is set to 0.5%. Additionally, the D-LS algorithm demonstrates strong computational performance in terms of optimality gap, achieving an average of 1.06% and a worst-case gap of 3.25% across all instance groups.

Table 1 – *Number of solved instances comparison of the solver and D-LS algorithm on all instances. The optimality gap is computed as* |`best_bound-objective_value`|/|`objective_value`|

# Stations	# Zones	# Distributions	# Solved Instances: Solver	D-LS	D-LS (Gap=0.5%)	#Optimality Gap: Solver	D-LS
15	3	125	25/36	36/36	36/36	3.30%	0.00%
24	3	125	15/36	32/36	32/36	-	0.07%
35	3	125	11/36	12/36	30/36	-	0.85%
Zones =3 Avg.			**51/108**	**80/108**	**98/108**	-	**0.31%**
15	4	625	13/36	35/36	35/36	-	0.09%
24	4	625	8/36	26/36	29/36	-	0.97%
35	4	625	0/36	14/36	21/36	-	2.35%
Zones =4 Avg.			**21/108**	**82/108**	**85/108**	-	**1.13%**
15	5	3125	5/36	34/36	34/36	-	0.10%
24	5	3125	0/36	17/36	21/36	-	1.92%
35	5	3125	0/36	15/36	17/36	-	3.25%
Zones = 5 Avg.			**5/108**	**66/108**	**72/108**	-	**1.75%**
Avg.			**23.77%**	**70.37%**	**78.70%**		**1.06%**

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