Line Network Design for Parcel Routing with Handling Times

Gabriel Deza, Michal Tzur, Tal Raviv
Department of Industrial Engineering, Tel Aviv University

Extended abstract submitted for presentation at the 12th Triennial Symposium on Transportation Analysis conference (TRISTAN XII)
June 22-27, 2025, Okinawa, Japan

March 27, 2025

Keywords: Parcel Delivery, Line Planning, Consistent Routing, Hyperconnected Network Design

1 INTRODUCTION

As same-day delivery options become more common on e-commerce platforms, customers are expanding the range of items they purchase, as well as increasing the frequency and quantity of their orders. Many retail stores, whether local or part of large chains, face challenges in operating such delivery systems independently, as they lack the demand to justify the high operational costs and suffer from low consolidation opportunities and low vehicle occupancy rates.

A practical solution is to develop a network of micro-hubs, also known as service points (SPs), served by a fleet of vehicles. Retail stores can drop off their deliveries at the nearest SP, and the parcels are then routed through the system to the SP closest to the customer. Each vehicle in the fleet repeats a designated route, loading and unloading parcels at specific SPs along the way. The network is said to be hyperconnected, as the SPs may also act as intermediate storage points for in-transit parcels, enabling multi-leg delivery routes.

These systems are sometimes referred to as *Physical Internets* or *Hyper-connected Networks* (HCNs). The latter emphasizes transshipment opportunities, in contrast to *Tree-like Service Networks* (TLSNs), where all demand must be routed up a hierarchy to a central depot—usually on the outskirts of a city—before routing down the hierarchy to its final destination. In TLSNs, all parcels from the same origin-destination pair have only a single delivery option, which does not fully utilize transshipment opportunities.

As more retail stores participate and contribute to the system through fees, operational costs are shared. Such a collective approach generates sufficient demand, enabling the network to benefit from consolidation opportunities and high vehicle occupancy rates. Pilot programs and startups, such as Homerr and Metapack in the Netherlands Homerr (2023), Velove in Sweden Velove (2024), and Kiezbote in Berlin Seeck (2021), have already begun implementing these systems in urban areas across several European countries.

The operator of such a delivery system needs to determine the fixed periodic vehicle routes along with parcel routes so as to minimize the expected average delivery time. While breaking down the parcel routes into multiple legs allows better consolidation, it also increases the handling time of the parcels at the SPs and may cause further delays to other parcels in the system. Optimizing the system requires careful consideration of this tradeoff, especially in settings where:

1. Parcels must be loaded and unloaded sequentially

2. the loading and unloading times of a single parcel are non-trivial compared to the travel times between micro-hubs

3. Drivers may end up idling a significant portion of their day due to handling parcels at SPs

Our work aims at determining the consistent vehicle routes, parcel routes, and handling times at each SP for each vehicle that will result in the minimal average parcel delivery time.

2 METHODOLOGY

In the following section, we first begin with a formal description of our problem. We then describe the construction of an extended network inspired by the line planning literature for metro and bus systems. We end with a brief description of our solution method.

2.1 Problem Description

Consider a network represented as a complete directed graph $G = (N, A)$, where N is the set of SPs in which demand originates and are destined to, and A is the set of potential arcs with travel times c_a for arc $a = (i, j) \in A$. G is referred to as the Physical Transport Network (PTN). Let λ_{ij} be the mean arrival rate of parcels originating at SP i destined to SP j. The network has a budget of k vehicles with each one being assigned to a line. A *line* is defined as a simple cycle (cycle where each node is visited once). A *line concept* refers to an allocation of k vehicles to a subset of all possible lines in G. The vehicles service the lines repeatedly with a fixed visit frequency to each SP along the line. Parcels travel along these lines to their destination, potentially transferring between them by being stored intermediately at an SP. Once a vehicle arrives to an SP it stops to potentially unload parcels off and load waiting parcels onto the vehicle. The time to load and unload a single parcel is t_l and t_u, respectively. The time a vehicle of a line stops at an SP along that line is called *service time*.

Our problem consists of *selecting* the line concept that minimizes the average parcel delivery time. While our goal is to determine the optimal line concept, we must also define (and optimize) auxiliary decisions for the parcel routes and service times so that the average parcel delivery time can be well defined. We aim to solve three problems jointly: determining (1) an optimal line concept, (2) parcel routes over the network induced by the line concept, and (3) the service times for each stop in the line concept.

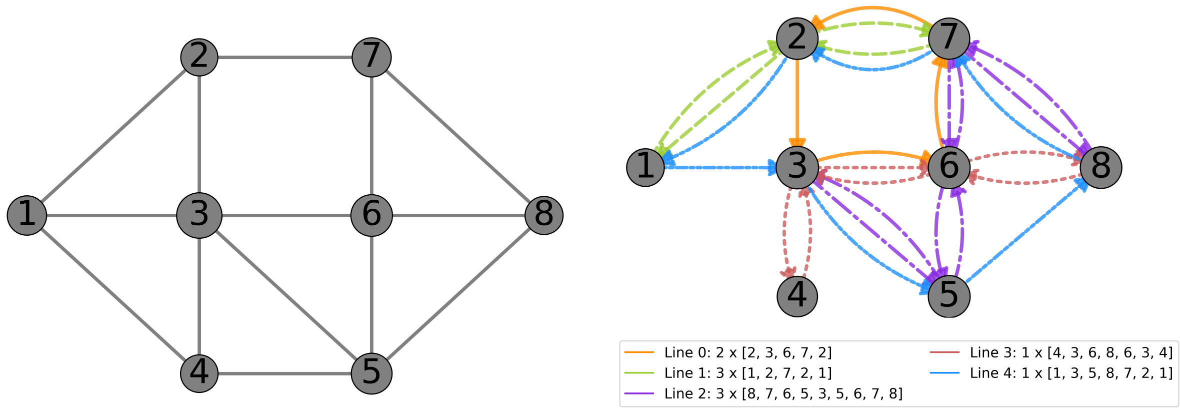
To distinguish between sub-problems, we will refer to the optimization over line concepts as **line planning**, optimization parcel policies as **parcel routing**, and optimizing service times as **vehicle scheduling** problems.

For illustration, Figure 1a shows an eight node PTN. The sequence of SPs $[2, 3, 6, 7, 2]$ is an example of a simple cycle forming a line. An example of a simple path back and forth forming a line is $[1, 2, 7, 8, 7, 2, 1]$. Given a PTN, the set of all lines is referred to as the line pool $\mathcal{P}$. The PTN in Figure 1a has 322 possible lines. The number of line concepts is then $\binom{|\mathcal{P}|+k-1}{|\mathcal{P}|-1}$ which equals $\approx 3.79 \times 10^{18}$ for this toy instance. An example of a line concept over the PTN in Figure 1a with a budget of 10 vehicles is shown in Figure 1b. The line concept consists of 5 unique lines, some having multiple vehicles assigned to them.

2.2 Extended Network

Formally, a line concept $\mathcal{L} \coloneqq (L, \{a_l\}_{l \in L})$ consists of a subset of lines L of the line pool $\mathcal{P}$, and the number of vehicles associated to each line l denoted f_l. The total number of assigned vehicles $\sum_{l \in L} f_l$ must equal the budget k. Our goal is to create an extended network that models the cost associated with handling parcels as well as transferring between lines at any transshipment opportunity.

Every time a parcel is loaded onto a vehicle, it requires t_l time and t_u to be unloaded off a vehicle. We assume a vehicle stays at an SP for a fixed time at every visit, denoted its *service time*. The service time of a vehicle of line l arriving at an SP i along l is represented as Δ_i^l. This time is dedicated to all possible loading and unloading of parcels. Δ_i^l is a decision variable to



(a) *PTN* (b) *Line concept with 4 lines and 10 vehicles*

Figure 1 – *PTN and example line concept of an eight node network.*

be optimized for the vehicle scheduling sub-problem. This assumption ensures that the system operates on a fixed periodic schedule, allowing the operators to know when each vehicle arrives at an SP along their line.

We can now define an extended directed network $H = (U, K)$ based on the PTN $G = (N, A)$ and $\mathcal{L}$. For every SP $i \in N$ create the node IO_i ("IO" indicating where parcels 'Input/Output' from). For every SP i along a line l for all lines in L, create a node V_i^l ("V" indicating it is related to a vehicle arrival to an SP). For every node i in line l for all lines $l \in L$, add the following three edges and their costs, measured in time units:

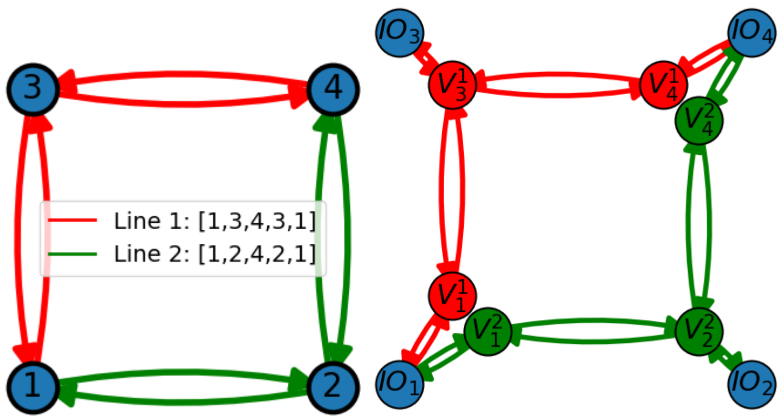
- **Transfer Arcs:** The expected transfer time is the expected time required for a vehicle of line l to visit an SP. Given f_l vehicles in line l, the expected transfer time is $\frac{\sum_{ij \in L}(c_{ij}+\Delta_i^l)}{2f_l}$.
- **Unloading arcs:** Once a vehicle of line l arrives to an SP i, it stays Δ_i^l time to load and unload parcels. In the case that a parcel "wants" to be unloaded, we do not know if it will be unloaded first, last, or anything in between. In the worst case, being loaded/unloaded last, it will have to wait Δ_i^l. This will be the cost of an unloading arc.
- **Travel arcs:** Upon arrival of a vehicle of line l to an SP i, the vehicle stays Δ_i^l during which it unloads and loads parcels. During this time, all parcels onboard also wait. Hence, the resulting traveling from SP i to j along line l is $c_{ij} + \Delta_i^l$.

Figure 2a showcases a four node grid network with 2 lines shown in red and green. The resulting extended network is shown in Figure 2b. In the extended network, a parcel with origin 3 and destination 2, via taking the red line from 3 to 1 and then transferring to the green line at SP 1, and then traveling to its destination node 2 can be modeled via all 3 arc types. Specifically, the path would be $IO_3 \to V_3^1 \to V_1^1 \to IO_1 \to V_1^2 \to V_2^2 \to IO_2$.

2.3 Solution Method

To solve our problem, we devise a generalized multicommodity flow over the extended network. Specifically, given the line pool $\mathcal{P}$ as a line concept, we construct the extended network and solve a non-convex Quadratically Constrained Quadratic Program (QCQP) that optimizes over the following decision variables:

- f_l (integer): the number of vehicles assigned to line $l \in \mathcal{P}$
- x_a^k (continuous): the flow of commodity k on arc $a \in K$ (the extended network)



(a) *Line Concept* (b) *Extended Network representation*

Figure 2 – *2 line Line concept on a 2 by 2 grid network and its extended network representation*

- Δ_i^l (continuous): the service time of a vehicle of line l stopping at SP i

This model jointly solves the line planning, parcel routing, and vehicle scheduling problems to determine the line concept that minimizes the expected average parcel delivery time, along with the associated parcel routes and service times. In addition, we devise a guided large neighbor search method over the space of all possible line concepts to overcome some of the challenges of solving this QCQP. Specifically, we modify the QCQP by fixing the line planning variables (f_l) and using the model as a black box to solve the parcel routing and vehicle scheduling aspects, which then guide a search over the space of all possible line concepts.

3 RESULTS

Numerical experiments are done on a wide range of both synthetic and realistic test beds. In all instances, we test the exact model and the guided large neighbor search procedure to find an optimal or good candidate line concept. While we find that in trivially small instances, modern solvers can solve the non-convex QCQP faster than the heuristic method proposed (and at higher precision), any non-trivial setting our search procedure is able to provide better line concepts than the exact method given the same time budgets, and with testing a small number of candidate line concepts.

Lastly, additional experiments are conducted to evaluate the strategic choice between using Hyperconnected Networks and traditional Tree-Like Service Networks. While it's clear that allowing transshipment requires re-optimizing parcel flows to reduce delivery times, we also find a significant benefit in re-optimizing vehicle routes. This adjustment further improves delivery efficiency, taking full advantage of transshipment opportunities.

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