A Unified Approach to Evaluation and Routing in Public Transport Systems

Rolf N. van Lieshout[1]
Kevin Dalmeijer[2]

[1]Department of Industrial Engineering Innovation Sciences, Eindhoven University of Technology, Eindhoven, the Netherlands
[2]H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology, United States

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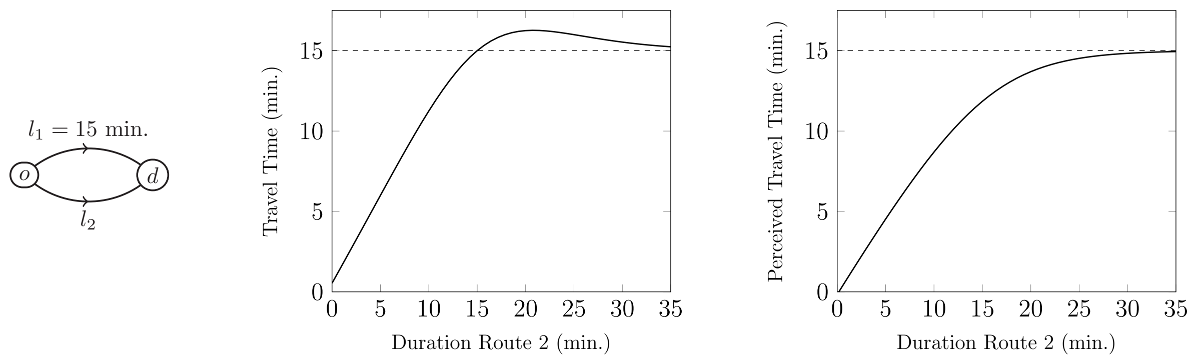
1 INTRODUCTION

In passenger mobility research, it is common to define some measure for the service quality of a public transport system (e.g., average travel time) and to assume some choice model that distributes travelers over routes and/or modes (e.g., the multinomial logit model). Despite their ubiquity, there is currently no consensus on *how* the service quality of public transport should be measured and *what* choice model is appropriate for what context. Instead, the literature offers a variety of approaches, which makes it difficult to compare and validate obtained results. Furthermore, it is generally accepted that public transport systems can improve service quality by adding routes, making routes faster or increasing the frequency at which routes are operated (at least, in the absence of congestion effects). However, this is not always reflected in existing approaches, leading to counter-intuitive and inconsistent results.

To illustrate how seemingly sensible route choice models and service quality measures can lead to unexpected outcomes and suboptimal decisions, suppose that travelers choose routes according to the logit model and that average travel time is used as a measure of service quality. For the system depicted in Figure 1a, where travelers can choose between route 1 with a duration of 15 minutes and route 2 with duration of l_2 minutes, Figure 1b shows the average travel time as a function of l_2. After some point, making route 2 slower actually *improves* the measure, because it increases the likelihood that travelers switch to route 1, which is better in terms of travel time. Public transport planners using such a measure are insufficiently incentivized to speed up or add routes. In fact, they may even decide to slow down or remove routes, as it could occur that their measure suggests that doing so improves service quality. Note that this phenomenon is fundamentally different than Braess' paradox, because in our setting travel times are fixed, whereas in Braess' paradox travel times depend on the number of travelers that choose a route (Braess, 1968).

In this paper, we present a formal treatment of route choice models and service quality measures for three supply models of public transport: route sets, (periodic) timetables and line plans, concepts illustrated in Figure 2. Throughout the paper, we consider a single origin destination



(a) *Routes.* (b) *Travel Time as function of* l_2*.* (c) *Perceived Travel Time as function of* l_2*.*

Figure 1 – *Illustration of two route set measures, where travelers are distributed over the routes using the logit model with* $\beta = 0.22$*. The duration of route* i *is denoted by* l_i*.*

(OD) pair, but this can be generalized to multiple OD pairs by taking a weighted average. We analyze the two predominant route choice models in the literature: *shortest path* routing, where all travelers choose the shortest route, and *logit* routing, where travelers distribute over the routes according to the logit model (Ben-Akiva & Lerman, 1985, McFadden, 1974). We then define desirable properties of measures, show which measures fail to meet these properties and develop measures that do. To ensure that the derived measures are consistent and interpretable, our approach is hierarchical: the line plan measures build upon the timetable measures, which again build upon the route set measures. Ultimately, this results in a ready-to-use framework for routing and evaluation in public transport, as well as multiple managerial insights.

2 FRAMEWORK

This section provides a high-level overview of our approach and results. A route set is simply a set of routes with given durations (see Figure 2a). A measure for route set quality can be constructed by pairing a routing model, e.g. logit, with an evaluation function, e.g. travel time. The desirable properties that we define are *monotonicity* and *consistency*. A measure is monotonic if increasing route durations or removing a route cannot improve the measure. Figure 1b illustrates that the combination of logit routing and travel time evaluation induces a measure that fails to be monotonic. We say a measure is consistent if the routing model minimizes the evaluation function. Again, travel time under logit routing fails to satisfy this property, since a shortest path routing will always lead to a shorter travel time compared to logit routing.

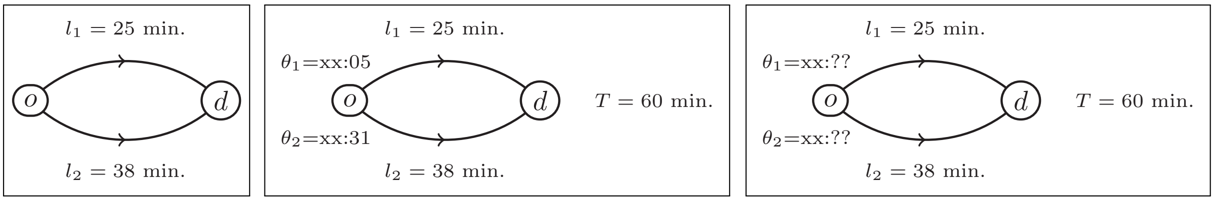
The observation that logit routing does not minimize travel time raises the question if there exists an alternative evaluation function that is minimized by logit. We answer this question affirmatively, by showing that logit routing optimizes the evaluation function we refer to as *perceived travel time*. This results in a measure that is both consistent and monotonic. The monotonicity can also be observed in Figure 1c, which depicts perceived travel time under logit routing. Moreover, we provide a motivation for the terminology “perceived travel time” through the random utility model (RUM) interpretation of logit (Train, 2009). In addition to evaluating perceived travel time under logit routing, combining regular travel time evaluation with shortest path routing also induces a consistent and monotonic measure.

To also support practitioners in other stages of the public transport planning process, this paper subsequently extends the analysis from route sets to periodic timetables. A periodic timetable is a route set that is operated periodically with a given cycle time or period and with given departure times (see Figure 2b). The time until a traveler reaches its destination now not only depends on the route durations, but also on the waiting time that the traveler experiences. Similar to for example Kaspi & Raviv (2013) and Polinder *et al.* (2021), we assume travel demand is distributed uniformly over the period. The reason for this is twofold:

1. Service quality should be measured with respect to the actual (unobserved) demand rather than the induced demand. Of course, in practice travelers plan their arrival to the station according to the timetable. However, their *desire to travel* is in many cases approximately uniform.

2. Timetables with equidistant departures (e.g., at xx:00, xx:15, xx:30 and xx:45) are typically preferred over timetables where departures are clustered (e.g., at xx:00, xx: 05, xx: 50, xx:55). Uniformly distributed travel demand naturally captures this intuition, since expected waiting time is minimized by equidistant departures.

Under uniform arrivals, we show how to efficiently compute the average travel time under shortest path routing, and the average perceived travel time under logit routing, resulting in two consistent and monotonic measures for the quality of a periodic timetable.



(a) *Route Set.* (b) *Timetable.* (c) *Line Plan.*

Figure 2 – *Illustration of the differences between a route set, timetable and line plan.* l_i *and* θ_i *represent the route duration and departure time of route* i*, respectively.* T *is the cycle time.*

Finally, we consider line plans, which are route sets that will be performed periodically, but with unknown departure times (see Figure 2c). Line plans are commonly used in strategic planning, when the timetable is not yet known (Schöbel, 2012). There, one needs measures for service quality and route choice models that are independent of the timetable that will be operated. To deal with this additional source of uncertainty, we construct line plan measures by optimizing the respective timetable measures over all possible timetables. Surprisingly, we are able to prove that the solution to this optimization problem can be interpreted as a routing, such that we obtain the route choice model *for free:* one simultaneously finds an estimate of service quality (the optimal value) and a routing (the optimal solution itself). We additionally develop algorithms that efficiently solve this optimization problem both for shortest path and for logit routing.

3 PRACTICAL INSIGHTS

The developed framework reveals a deep parallel between evaluation and routing: by solving an appropriate optimization model one obtains both the quality of the route set, timetable or line plan (the optimal objective value), and the distribution of the travelers over the routes (the optimal solution itself). Hence, every evaluation function *implies* some route choice model, and vice versa. The importance of this connection for public transport planners is evident for route sets from the example in Figure 1, but this behavior extends all the way to line plans: situations where the evaluation function and routing do not correspond lead to missed opportunities for improving services and taking wrong decisions that can harm the experience of public transport users.

Further numerical experiments disclose more important insights for practitioners. Route choice and service quality can differ substantially between route sets, timetables, and line plans, highlighting that it is crucial to select the right model at the right stage of planning. Furthermore, while logit is the model of choice for accurately capturing traveler behavior, the benefit of logit over shortest path diminishes as we move from route sets to timetables to line plans. This observation has major managerial implications, as it implies that *it is not always necessary for public transport planners to accurately model travel behavior*. Especially for long-term or strategic planning such as line planning, the shortest path model may be sufficiently accurate to make good decisions.

4 CONCLUSION

Concluding, the main contribution of this paper is twofold. Firstly, we develop and analyze a framework for traveler route choice and service quality evaluation for route sets, timetables and line plans. Compared to existing work, the framework introduces a new evaluation function that is consistent with the logit model, and routing models and measures tailored to the specific nature of line plans. Secondly, based on the framework we derive and discuss several managerial implications. Among other insights, we demonstrate using numerical experiments that the framework developed in this paper enables planners to (i) improve service by taking better decisions and to (ii) assess the difference between shortest path and logit measures for their network, avoiding investments in complicated methods when they are unnecessary.

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