Mean-Field Game Optimization in Bounded-Acceleration Traffic Models for CAVs

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1 INTRODUCTION

In recent years, the deployment of Connected Autonomous Vehicles (CAVs) has highlighted the need for advanced traffic models capable of handling dynamic, high-density conditions while ensuring safety and efficiency. Traditional traffic flow models struggle to accommodate the complexities of CAV dynamics, particularly under conditions of bounded acceleration, where sudden changes in speed and traffic density can lead to significant inefficiencies and congestion. Bounded-acceleration traffic flow (BATF) models have been developed to address the limitations of traditional models, particularly in representing traffic speed dynamics and accounting for bounded acceleration Lebacque (2002). Traditional traffic models often lack the flexibility to account for dynamic changes in traffic conditions and driver behavior, especially under partial congestion or during rapid acceleration and deceleration phases. This results in an inability to accurately model traffic in scenarios with a high degree of variability, which is increasingly relevant with the adoption of Connected Autonomous Vehicles (CAVs). These models are especially relevant with the rise of CAVs, as they can be designed to optimize traffic flow through controlled acceleration. By introducing bounded-acceleration constraints, BATF models aim to improve safety, efficiency, and predictability in vehicle interactions, particularly in complex urban and highway environments where unexpected slowdowns and acceleration may occur. Besides, they are effective in partially congested traffic scenarios and, with a control scheme, regulate traffic flow by controlling vehicle acceleration Jin & Laval (2018).

Several studies have explored bounded-acceleration models to better capture traffic dynamics. Qiu *et al.* (2013) provided grid-free solutions to the Lighthill-Whitham-Richards (LWR) traffic flow model with bounded acceleration. Lebacque (2003) introduced a two-phase model distinguishing between equilibrium and bounded-acceleration phases. Leclercq (2007) and Jin & Laval (2018) further refined these models, providing a unified approach to BATF. Recently Mean-Field Game (MFG) frameworks offer promising foundations for optimizing vehicle behavior in large systems but have not yet been fully adapted to the specific challenges posed by bounded-acceleration traffic models. For example, Ameli *et al.* (2022) applied the MFG framework to urban transportation, demonstrating its potential in optimizing departure time choices.

This study addresses the pressing question: How can a MFG framework be effectively tailored to bounded-acceleration traffic models to optimize CAV trajectories and improve overall traffic flow? We propose a novel approach that incorporates MFG within the constraints of bounded acceleration, enabling a more adaptive and realistic optimization method for CAV traffic systems. By designing an MFG framework that can respond to real-time variations in traffic density and vehicle dynamics, this research contributes a significant advancement in traffic flow management for CAVs. Specifically, the proposed MFG model optimizes vehicle trajectories in a manner that enhances system-wide efficiency, mitigates congestion, and supports the safe integration of CAVs into urban and highway networks.

2 MODEL FORMULATION

In bounded-acceleration models, traffic flow is divided into two phases: equilibrium (EQ) and bounded-acceleration (BA). The transitions between these two phases considering the trsffic model of Lighthill–Whitham–Richards (LWR) is detailed in Lebacque (2003). The fundamental equations for the conservation of vehicles and momentum are:

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2) = \rho a(\rho, u), \end{cases} \tag{1}$$

where ρ represents density, u is velocity, and $a(\rho, u)$ is the acceleration, which is bounded by a maximum value A. Therefore, the cost functional for each vehicle i is defined as:

$$J_i(u) = \int_0^T \left[L(\rho(t, x_i(t)), u(t, x_i(t))) + \frac{1}{2}\|u(t)\|^2 \right] dt, \tag{2}$$

where L is a Lagrangian representing the driving cost.

In traditional bounded-acceleration traffic flow models, the acceleration a is often defined as a fixed function of vehicle density and velocity constraints. However, we adopt an alternative approach by redefining acceleration within a set $K(\rho, u)$ rather than as a strictly bounded parameter. This new formulation allows for a more flexible control mechanism, optimizing vehicle trajectories and accommodating various traffic dynamics, especially within Connected Autonomous Vehicle (CAV) systems. Rather than explicitly bounding acceleration through a fixed function, we define a as a variable constrained by:

$$a \in K(\rho, u),$$

where the set $K(\rho, u)$ is determined by the relationship between the velocity u and the equilibrium velocity $u_e(\rho)$, given as:

$$K(\rho, u) = \begin{cases} [0, A] & \text{if } u < u_e(\rho), \\ [-\beta A, 0] & \text{if } u > u_e(\rho), \\ [-\beta A, A] & \text{if } u = u_e(\rho), \end{cases} \tag{3}$$

where A is the maximum allowable acceleration and βA represents the maximum allowable deceleration. This formulation addresses potential inconsistencies that arise from simultaneously defining acceleration as both the derivative of velocity in the optimal trajectory problem (Eq. 2) and as a fixed function. By eliminating the dual definition of a, this approach also increases the model's adaptability and alignment with real-world traffic dynamics.

With this redefined acceleration set, we reformulate the optimal control problem (Eq. 2) as follows:

$$\begin{array}{l} \min_a \int_0^T \left(L(\rho, u) + \frac{a^2}{2} \right) dt \\ \left| \begin{array}{l} \dot{x} = u, \\ \dot{u} = a, \\ a \in K(\rho, u), \\ \text{Initial conditions for } x \text{ and } u, \end{array} \right. \end{array} \tag{4}$$

where x and u are the position and velocity of the vehicle, respectively, and a is the control variable constrained within $K(\rho, u)$. This formulation ensures positive vehicle speeds due to the constraints defined by $K(\rho, u)$.

To solve this optimal control problem, we define the Hamiltonian H for the system:

$$H(\rho, u, a, p, q) = L(\rho, u) + \frac{a^2}{2} + p \cdot u + q \cdot a, \tag{5}$$

where p and q are adjoint variables. We then derive the reduced Hamiltonian H_0 by minimizing H with respect to a:

$$H_0(\rho, u, p, q) = \min_{a \in K(\rho,u)} H(\rho, u, a, p, q), \tag{6}$$

which yields:

$$H_0(\rho, u, p, q) = L(\rho, u) + \frac{a^2}{2} + p \cdot u + q \cdot a, \quad a = \Pi_{K(\rho,u)}(-q), \tag{7}$$

where $\Pi_{K(\rho,u)}(.)$ denotes the projection operator onto $K(\rho, u)$. The projection operator $p_{[a,b]}(y)$ is defined as:

$$p_{[a,b]}(y) = \begin{cases} a & \text{if } y \leq a, \\ b & \text{if } y \geq b, \\ y & \text{if } a \leq y \leq b, \end{cases} \tag{8}$$

allowing us to express $\Pi_{K(\rho,u)}(.)$ as follows:

$$\Pi_{K(\rho,u)}(y) = \begin{cases} p_{[0,A]}(y) & \text{if } u < u_e(\rho), \\ p_{[-\beta A,0]}(y) & \text{if } u > u_e(\rho), \\ p_{[-\beta A,A]}(y) & \text{if } u = u_e(\rho). \end{cases} \tag{9}$$

In this model, the Hamilton-Jacobi equation for the value function $V(x, u, t)$ is defined as:

$$\begin{aligned} &\partial_t V + H_0 (\rho, u, \partial_x V, \partial_u V) = 0, \\ &V|_{t=T} = 0, \end{aligned} \tag{10}$$

where V has a terminal value of zero due to the absence of a specific terminal criterion.

The coupled Partial Differential Equations (PDEs) in the MFG system therefore can be formulated for bounded-acceleration traffic flow is therefore reformulated as follows:

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2) = \rho a(x, t), \\ \partial_t V + H_0 (\rho, u, \partial_x V, \partial_u V) = 0, \\ a = \Pi_{K(\rho,u)} (-\partial_u V), \end{cases} \tag{11}$$

where the control variable is now the acceleration a, rather than the velocity u in the classical formulation and we retain the Eulerian form of the acceleration equation, where a depends on both x and t. (The dependence of a on u, a function of x and t, may also be necessary for full model fidelity).

The proposed MFG formulation captures the more complex acceleration dynamics of CAVs in partially congested conditions and enables more precise optimization of vehicle trajectories.

3 NUMERICAL EXPERIMENTS

In this section, we present preliminarily results of the numerical simulations to demonstrate the effectiveness of the proposed MFG framework in optimizing vehicle trajectories within bounded-acceleration traffic models. The implementation focuses on evaluating how the MFG model adapts to varying traffic densities and vehicle interactions, using a combination of fixed-point algorithms and a fictitious play technique to solve the coupled PDEs in Eq. 11. This iterative algorithm designed to converge to an ϵ-Nash equilibrium, ensuring that each vehicle's trajectory optimally responds to the collective movement of others. The fictitious play approach updates each agent (vehicle) strategy iteratively based on anticipated responses of the entire field of vehicles. To solve the MFG system's coupled PDEs numerically, we discretize the equations over

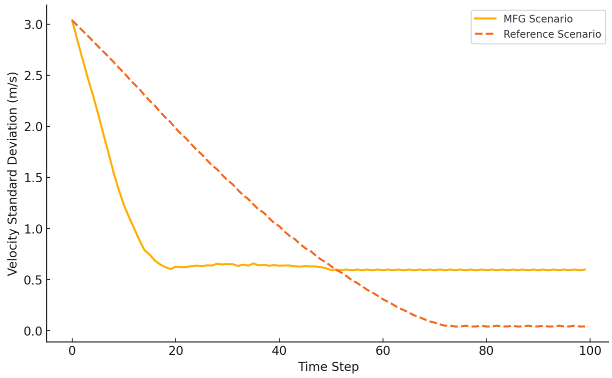

Figure 1 – *Comparison of Velocity Standard Deviation over time step*

a spatial-temporal grid, leveraging finite difference methods for spatial derivatives and explicit time-stepping. Each iteration in the fixed-point algorithm includes calculating the projected acceleration set $K(\rho, u)$ for each vehicle based on the local traffic density ρ and velocity u. By constraining acceleration within this set, we achieve realistic and bounded vehicle movements, crucial for CAV applications where safe, gradual speed changes are required.

The model is applied an urban traffic corridor simulated using parameters derived from the Next Generation Simulation (NGSIM) dataset I-80 study area, which provides high-resolution traffic flow data. We specifically selected segments with varied traffic density and congestion levels to observe the MFG framework's adaptability to different driving conditions. The simulations included morning peak traffic scenarios, during which rapid deceleration and congestion are common. Initial conditions were set to represent realistic starting positions, velocities, and acceleration bounds, consistent with observed urban traffic flows in the dataset. Preliminary results for velocity standard deviation, shown in Fig.1, indicate that the MFG framework optimizes vehicle trajectories more effectively than Baseline BATF models, yielding realistic and smoother traffic flow across varying densities. Initially, the MFG scenario achieves a lower standard deviation than the reference scenario, demonstrating reduced speed variability and smoother transitions, which help mitigate congestion. After time step 50, however, the reference scenario's standard deviation falls below that of the MFG, reflecting a late-stage stabilization. This suggests that while the baseline model may eventually stabilize, it lacks the MFG model's adaptive responsiveness, which dynamically adjusts trajectories based on real-time traffic changes. This continuous adaptability of the MFG framework enhances traffic flow uniformity and supports efficient traffic management, showing promise for real-time optimization in smart city infrastructure.

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