A Logic-Based Benders Decomposition Approach for Cyclic Microscopic Timetabling

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1 INTRODUCTION

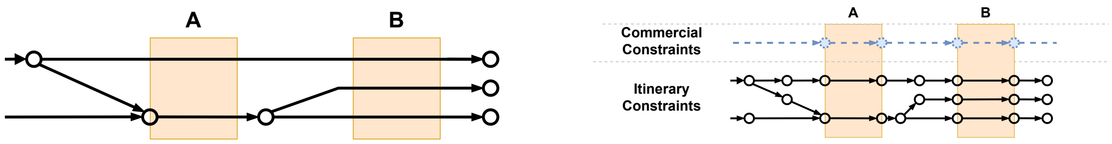
Timetabling in railway systems is a critical task that ensures trains' efficient and safe movement while meeting passenger demands and operational constraints. Traditionally, timetabling approaches are classified into macroscopic and microscopic levels and into cyclic (periodic) and non-cyclic (aperiodic) timetables. Periodic timetables are commonly used for passenger trains as they are convenient for passengers and infrastructure investments, whereas aperiodic timetables are more common for freight trains. At the macroscopic level, models like the Periodic Event Scheduling Problem (PESP) are widely used for generating cyclic timetables in large-scale strategic planning and support integrated planning such as passenger routing (Polinder *et al.*, 2021) and vehicle circulation (Lieshout, 2021). However, PESP does not guarantee feasibility at the microscopic level, where detailed infrastructure constraints are critical. Microscopic Railway Timetabling Problems (MRTPs), on the other hand, provide a detailed representation of the infrastructure, including tracks, signals, and block sections (Leutwiler & Corman, 2023). The MRTP ensures operational feasibility by optimizing infrastructure usage and conflicts. However, these models are complex and computationally intensive.

Few models attempt to find cyclic microscopic timetables for large-scale networks, and those typically employ heuristic or sequential methods, which do not guarantee optimality or feasibility (Bešinović *et al.*, 2016, Caimi *et al.*, 2017). We present an exact approach that leverages decomposition techniques to achieve microscopically feasible timetables at a large scale. Using real-world data from a Swiss rail network, the results suggest that our method outperforms traditional MRTP methods greatly thanks to our proposed aggregated cuts. Comparisons to macroscopic equivalent formulations indicate that our model achieves better solutions, providing operators with timetables that are strategically sound and operationally feasible at the microscopic level.

2 MODEL FORMULATION

We optimize the route and schedule of a set of periodic trains $l \in \mathcal{L}$. We are given all trains to schedule during one period T, so the timetable can be rolled out over multiple periods. We aim to minimize total running and dwelling time, a common objective in PESP (Lieshout, 2021).

To define the route choices of each train $l \in \mathcal{L}$ across the network, we use a *Train Flow Network* (TFN) (see Figure 1a) with nodes $n \in \mathcal{N}_l$ representing infrastructure decision points and arcs $w \in \mathcal{W}_l$ representing track segments between nodes. The binary variables $x_w \in \{0, 1\}$ indicate whether a train uses a particular link $w \in \mathcal{W}_l$.



(a) *Train Flow Network (TFN).*

(b) *Commercial and itinerary constraints in the Event Activity Network (EAN).*

Figure 1 – *Example of the network representations for a train.*

To schedule each train $l \in \mathcal{L}$ on a microscopic level, we use an *Event Activity Network* (EAN), where nodes refer to events $e \in \mathcal{E}_l$, each associated with a timestamp t_e. Arcs refer to activities $a \in \mathcal{A}$, representing relationships between events (e.g., running and dwelling times). The EAN is closely linked to the TFN, where each route in the TFN corresponds to a sequence of events and activities in the EAN. We classify the activities into itinerary, commercial, and periodicity activities. Itinerary activities ($\mathcal{A}^{\text{ITI}}$) represent the technical requirements, such as running and dwelling times across different routes. Commercial activities ($\mathcal{A}^{\text{COM}}$) capture service-related requirements, such as scheduled arrival and departure times, and connections for passenger transfers (e.g., see Figure 1b). Finally, periodicity activities ($\mathcal{A}^{\text{PER}}$) are used to ensure a cyclic timetable. These activities link the events across different periods, enforcing that services are repeated each period T and on the same route. Since some trains may take longer than one period T to complete their journeys, we need to roll out the timetable over $\hat{k} = \lceil \frac{\Delta}{T} \rceil + 1$ periods where Δ denotes the maximum train run time.

A key challenge in timetabling is ensuring that trains do not conflict (i.e., use the same infrastructure resource simultaneously). To prevent such conflicts, we model the occupation of resources by defining entry and exit events in the EAN for each train's use of a resource $p \in \mathcal{P}$ and their respective times $t_p^{\text{entry}}, t_p^{\text{exit}}$. We use binary variable z_p to indicate if resource path p is used by a train. To prevent conflicts between trains, we also define binary variables $y_{(p,p')}$ to indicate if the train using resource path p clears the resource before the train using resource path p' occupies it, for all conflicting pairs $(p, p') \in \mathcal{K}$.

Based on all these considerations, we present the Cyclic-MRTP (C-MRTP) model, which integrates the temporal and spatial periodicity of train operations while considering infrastructure usage and conflict avoidance. The model is formulated as follows:

$$\min \sum_{a=(i,j)\in\mathcal{A}^{\text{COM}}} (t_j - t_i) \tag{1a}$$

subject to:

$$\sum_{w\in\rho(n)^+} x_w = 1, \quad \forall l \in \mathcal{L}, n \in \mathcal{N}_l^{\text{Source}}, \tag{1b}$$

$$\sum_{w\in\rho(n)^+} x_w = \sum_{w\in\rho(n)^-} x_w, \quad \forall l \in \mathcal{L}, n \in \mathcal{N}_l \setminus (\mathcal{N}_l^{\text{Source}} \cup \mathcal{N}_l^{\text{Sink}}), \tag{1c}$$

$$lb_a \cdot x_{w(a)} \le t_j - t_i \le ub_a \cdot x_{w(a)} + M \cdot (1 - x_{w(a)}), \quad \forall a = (i,j) \in \mathcal{A}^{\text{ITI}}, \tag{1d}$$

$$lb_a \le t_j - t_i \le ub_a, \quad \forall a = (i,j) \in \mathcal{A}^{\text{COM}}, \tag{1e}$$

$$z_p \ge \sum_{w\in\mathcal{W}_p} x_w - |\mathcal{W}_p| + 1, \quad \forall p \in \mathcal{P}, \tag{1f}$$

$$y_{pp'} + y_{p'p} \leq 1, \quad \forall \{p, p'\} \in \mathcal{K}, \tag{1g}$$
$$y_{pp'} + y_{p'p} \geq z_p + z_{p'} - 1, \quad \forall \{p, p'\} \in \mathcal{K}, \tag{1h}$$
$$t_{p'}^{\text{entry}} - t_p^{\text{exit}} + \delta \geq M(y_{pp'} - 1), \quad \forall \{p, p'\} \in \mathcal{K}, \tag{1i}$$
$$t_p^{\text{entry}} - t_{p'}^{\text{exit}} + \delta \geq M(y_{p'p} - 1), \quad \forall \{p, p'\} \in \mathcal{K}, \tag{1j}$$
$$t_{e^{(k)}} = t_{e^{(0)}} + kT, \quad l \in \mathcal{L}, \forall e \in \mathcal{E}_l : (e^{(0)}, e^{(k)}) \in \mathcal{A}^{\text{PER}}, \quad \forall k \in \{0, 1, \ldots, \hat{k} - 1\}, \tag{1k}$$
$$x_w^{(k)} = x_w^{(0)}, \quad \forall l \in \mathcal{L}, w \in \mathcal{W}_l, \quad \forall k \in \{1, \ldots, \hat{k} - 1\}, \tag{1l}$$
$$t_e \in \mathbb{R}^+, \quad \forall l \in \mathcal{L}, e \in \mathcal{E}_l, \tag{1m}$$
$$y_{pp'}, y_{p'p} \in \{0, 1\}, \quad \forall \{p, p'\} \in \mathcal{K}, \tag{1n}$$
$$x_w \in \{0, 1\}, \quad \forall l \in \mathcal{L}, w \in \mathcal{W}_l. \tag{1o}$$

The objective function (1a) minimizes the overall duration of commercial activities, a well-established objective used by PESP models in railway timetabling. Constraints (1b)-(1c) enforce flow conservation to ensure that each train selects precisely one path from origin to destination where $\rho(n)^+$ and $\rho(n)^-$ are the sets of outgoing and incoming arcs at node n, respectively. Constraints (1d) ensure that itinerary activities are only enforced when the corresponding route is selected where M is a sufficiently large constant, and Constraints (1e) enforce the time bounds for commercial activities. Constraints (1f) ensure a resource path is activated only if the train route uses a link of such path. We define Constraints (1g) to ensure that conflicting precedence conditions cannot hold simultaneously, and Constraints (1h) to enforce that the precedence constraints are only enforced when both resource paths p and p' are used. Constraints (1i)-(1j) ensure the gap between the trains' entry and exit times on conflicting resource paths respects the separation time δ. Constraints (1k) and (1l) define the temporal and spatial periodicity of the schedule and route choices, respectively. For simplicity, we omit the period index in the variables x_w and t_e in all remaining constraints as they represent the base case of $k = 0$. Finally, Constraints (1m)-(1o) define the nature of the decision variables.

3 A LOGIC-BASED BENDERS DECOMPOSITION METHOD

Even without periodicity, microscopic timetabling is a complex problem to solve. One promising approach to handle this issue is using decomposition methods (Leutwiler & Corman, 2022) such as logic-based Benders decomposition (LBBD). Exploiting the periodicity of the problem and the itinerary activities as feasibility constraints, we present an LBBD method, where the C-MRTP is split into a master optimization problem formed by Equations (1a),(1e) and (1m), and a feasibility sub-problem comprising the full C-MRTP comprised in (1) without the objective function. Solving the master problem yields a valid lower bound and a candidate solution for the subproblem. If any constraints are violated in the subproblem, we generate logic-based Benders cuts to update the master problem. Additionally, identifying these cuts provides a feasible solution as an upper bound. We iteratively solve the master and subproblem, adding cuts until convergence.

We propose to aggregate the cuts for LBBD in MRTP of Leutwiler & Corman (2022):

$$\bigvee_{d \in \mathcal{D}} \left(\sum_{a=(i,j) \in A_d} t_j - t_i \geq \delta_d \right), \quad \forall \mathcal{D} \in \mathcal{C} \tag{2}$$

where $\mathcal{D}$ represents a set of disjunctions, and for each disjunction $d \in \mathcal{D}$, there is a corresponding set of activities A_d that must be satisfied. The expression $\sum_{a \in A_d}(t_j - t_i)$ sums the time differences between the events i and j for all activities $a = (i, j) \in A_d$, ensuring that the total separation across all relevant activities is at least δ_d, the minimum required separation for disjunction d. The disjunction $\bigvee_{\mathcal{D}\in\mathcal{C}}$ ensures that at least one set of aggregated separations holds. While the cuts from Leutwiler & Corman (2022) focus on cutting conflicts related to individual events, our approach focuses on the activities between events. It allows us to aggregate several conflicting constraints into a single cut, eliminating infeasible solutions more efficiently.

4 Computational Results

We applied our model to a Swiss railway network operated by Rhätische Bahn (RhB), consisting of 14 lines (28 trains/hour), 102 stations, and 385 km of single- and multi-track sections. Five instances were generated, representing 20% to 100% of lines; each smaller instance is a subset of the larger. We compared three models: Direct Model (MIP), Logic-Based Benders Decomposition (LBBD) with the cuts from Leutwiler & Corman (2022), and our proposed method with aggregated cuts (LBBD-AGG). All runs were warm-started with a heuristic solution to ensure comparability in case of timeouts (5 h). Each instance was solved three times per method on four cores with GUROBI 11.0. Table 1 summarizes the average performance for each approach.

Network	**MIP**		**LBBD**				**LBBD-AGG**			
Size	Time (s)	Gap	Time (s)	Gap	Iterations	Cuts	Time (s)	Gap	Iterations	Cuts
20 %	2.4	0.0	2.7	0.0	10.3	276	0.6	0.0	4.0	47.0
40 %	106	0.0	15	0.0	15.7	1244	1.9	0.0	4.7	113.3
60 %	18000	4.3	216	0.0	17.0	4805	26	0.0	13.0	239.0
80 %	18000	5.5	4993	0.0	27.7	9362	749	0.0	58.3	453.7
100 %	18000	6.8	8147	0.0	30.0	10416	989	0.0	67.3	498.7

Table 1 – *Comparison of MIP, LBBD, and LBBD-AGG approaches across instances.*

The results confirm that instances with more trains are generally harder to solve. Among the methods, direct models fail to scale to instances of medium size, whereas decomposition-based methods can solve the largest instances. The proposed LBBD-AGG is consistently faster than LBBD, thanks to the effectiveness of the novel cuts. To assess the practical benefits of a microscopic approach, we also solved a macroscopic equivalent in the entire network, yielding a 1.5% increase in objective value. This translates to an additional 4.45 h over 10 h of operations, which can be saved using a microscopic model. Overall, our results demonstrate that the proposed LBBD-AGG approach is computationally efficient for large-scale, complex timetabling instances and highlights the potential advantages of adopting a microscopic model for periodic timetabling. In further research, we want to focus on tackling even more extensive networks and integrating practically relevant aspects, such as passenger routing, flexible transfers or vehicle circulation.

References

Bešinović, Nikola, Goverde, Rob M. P., Quaglietta, Egidio, & Roberti, Roberto. 2016. An integrated micro-macro approach to robust railway timetabling. *Transportation Research Part B: Methodological*, **87**, 14–32.

Caimi, Gabrio, Kroon, Leo, & Liebchen, Christian. 2017. Models for railway timetable optimization: Applicability and applications in practice. *Journal of Rail Transport Planning & Management*, **6**(4), 285–312.

Leutwiler, Florin, & Corman, Francesco. 2022. A logic-based Benders decomposition for microscopic railway timetable planning. *European Journal of Operational Research*, **303**(2), 525–540.

Leutwiler, Florin, & Corman, Francesco. 2023. A review of principles and methods to decompose large-scale railway scheduling problems. *EURO Journal on Transportation and Logistics*, 100107.

Lieshout, Rolf N. Van. 2021. Integrated periodic timetabling and vehicle circulation scheduling. *Transportation Science*, **55**, 768–790.

Polinder, Gert Jaap, Schmidt, Marie, & Huisman, Dennis. 2021. Timetabling for strategic passenger railway planning. *Transportation Research Part B: Methodological*, **146**, 111–135.