A new MILP formulation and a Branch-and-cut Algorithm for the TSP with Release Dates and Drone Resupply

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1 INTRODUCTION

Drones are increasingly used in last-mile logistics to streamline the final, often complex, delivery stage. While synchronized truck-and-drone systems, where trucks act as mobile depots, have been widely studied (Boccia *et al.*, 2023), direct drone deliveries face regulatory and safety challenges. An emerging alternative is using drones to resupply delivery trucks in transit, bypassing some regulatory barriers and enhancing efficiency without direct-to-customer deployment (Dayarian *et al.*, 2020). This model reduces reliance on centralized depots, improves delivery speed, supports real-time order fulfillment, and enhances flexibility, cost-effectiveness, and sustainability. Drones can deliver fresh packages to trucks on the road, reducing detours and fuel usage, especially in rural areas, while efficiently meeting fluctuating demand.

A system with a single truck receiving new orders en route via drone from a depot is defined as the Traveling Salesman Problem with Release Dates and Drone Resupply (*TSPRD-DR*). Research on this topic is still limited. It was first introduced by Dayarian *et al.* (2020) in a same-day delivery context and subsequently approached by a simulation method by Moshref-Javadi *et al.* (2023). However, the formalization and modeling of the *TSPRD-DR* were provided by Pina-Pardo *et al.* (2021), who later extended the framework to encompass vehicle routing and dynamic scenarios in Pina-Pardo *et al.* (2024b) and Pina-Pardo *et al.* (2024a), respectively. In particular, in their first study, Pina-Pardo *et al.* (2021) present the *TSPRD-DR* as a variant of the *TSPRD* introduced by Archetti *et al.* (2018), whereby a vehicle can return to the depot multiple times for resupply, thus allowing it to execute multiple routes to service end customers. They propose a mixed-integer linear programming (MILP) formulation alongside a decomposition-based heuristic approach for larger instances. However, the proposed MILP model suffers from dimensional drawbacks due to the presence of big-M constraints, rendering it ineffective even for small instances. In this work, we propose a novel formulation for the *TSPRD-DR* (Section 2). Furthermore, we introduce a Branch-and-Cut (*B&C*) algorithm to solve the problem and present preliminary results demonstrating the effectiveness of our approach in comparison to the existing literature (Section 3).

2 PROBLEM DESCRIPTION AND MILP FORMULATION

The *TSPRD-DR* aims to find a minimum-time route for a single truck that must serve a set of customers and can be resupplied en route with new orders by a drone. We assume that each customer places at most one order per day and that the release dates of these orders are known at the start of the planning horizon. Orders can be loaded onto the truck either at the depot or via the drone while en route. Additionally, we assume that the drone can rendezvous with the truck only at customer locations, and we account for unloading and drone launching times when these meetings occur. The truck does not return to the depot during the day, and the drone must return to the depot before it can be deployed again for resupply. We assume that the drone has specified capacity and flight endurance limits, while the truck has infinite capacity.

Given these assumptions, the *TSPRD-DR* can be formulated on a complete graph $G = (V, A)$, where V is the set of nodes and A is the set of arcs. Specifically, V consists of the union of the set of customers C and the nodes s and t, representing the starting and ending depots, respectively. The release date for customer $i \in C$ is denoted by r_i, with $r_i \geq 0$. A truck travel time t_{ij} is associated with each arc $(i, j) \in A$. Additionally, each customer $i \in C$ has an associated drone travel time d_i, as the drone can only travel between the depot and each customer location. Let Dtl and Cap represent the drone flight endurance and maximum capacity, respectively. The time required for loading and unloading orders is denoted by s_{time}. Finally, we define the set $K = \{1, ..., k_{max}\}$ as the set of resupplies, where k_{max} is the maximum number of possible resupplies. Given this setting, the following variables are defined:
- $x^k_{ij}, \forall (i, j) \in A, k \in K$: equal to 1 if the arc (i, j) is traversed by the truck after the k^{th} resupply and before the next one; 0 otherwise;
- $y^k_i, \forall i \in C, k \in K$: equal to 1 if customer i is served by the truck after the k^{th} resupply and before the next one; 0 otherwise;
- $z^k_i, \forall i \in C, k \in K$: equal to 1 if the order of customer i is loaded onto the truck during k^{th} resupply; 0 otherwise;
- $\delta^k_i, \forall i \in V, k \in K \cup \{k_{\max} + 1\}$, equal to 1 if the k^{th} resupply occurs at node i; 0 otherwise;
- τ^k_{start} and $\tau^k_{\text{end}}, \forall i \in V, k \in K$: continuous variables indicating the departure and arrival time of the truck at a node/depot where the k^{th} and $(k + 1)^{\text{th}}$ resupply occurs, respectively.

On this basis, the *TSPRD-DR* can be modeled as follows:

$$minimize \quad \tau^{k_{max}}_{end} \tag{1}$$

$$\sum_{j \in V} x^k_{ij} - \sum_{j \in V} x^k_{ji} = \delta^k_i - \delta^{k+1}_i \qquad i \in C \; k \in K \tag{2}$$

$$\sum_{j \in V} x^1_{sj} = \delta^1_s \tag{3}$$

$$\sum_{j \in V} x^k_{jt} = \delta^{k+1}_t \qquad k \in K \tag{4}$$

$$y^k_i = \sum_{j \in V} x^k_{ij} \qquad i \in C \; k \in K \tag{5}$$

$$\sum_{k \in K} y^k_i = 1 \qquad i \in C \tag{6}$$

$$\sum_{i,j \in S} x^k_{ij} \leq |S| - 1 \qquad S \subset V \; k \in K \tag{7}$$

$$\tau^k_{end} = \tau^k_{start} + \sum_{(i,j) \in A} t_{ij} x^k_{ij} \qquad k \in K \tag{8}$$

$$\tau^k_{start} \geq \tau^{k-1}_{end} + S_{Time} \sum_{i \in C} \delta^k_i \qquad k \in K \setminus \{1\} \tag{9}$$

$$\tau^k_{start} \geq \tau^{k-1}_{start} + \sum_{i \in C} (d_i + S_{Time})\, \delta^{k-1}_i + \sum_{i \in C} (d_i + S_{Time})\, \delta^k_i \qquad k \in K \setminus \{1\} \quad (10)$$

$$\tau^k_{start} - \sum_{j \in C} (d_j + S_{Time}) \delta^k_j \geq r_i z^k_i \qquad i \in C \;\; k \in K \setminus \{1\} \quad (11)$$

$$\tau^1_{start} \geq r_i z^1_i \qquad i \in C \quad (12)$$

$$\delta^1_s = 1 \qquad (13)$$

$$\sum_{k \in K'} \delta^k_t = 1 \qquad (14)$$

$$\sum_{i \in V} \delta^k_i \leq 1 \qquad k \in K \quad (15)$$

$$\sum_{k \in k} \delta^k_i \leq 1 \qquad i \in V \quad (16)$$

$$\sum_{k \in k} d_i \delta^k_i \leq Dtl \qquad i \in V \quad (17)$$

$$\sum_{i \in C} z^k_i \leq Cap \qquad k \in K \setminus \{1\} \quad (18)$$

$$y^k_i \leq \sum_{l=1}^{k} z^l_i \qquad i \in C \;\; k \in K \quad (19)$$

$$y^k_i \leq \sum_{j \in V} \delta^k_j \qquad i \in C \;\; k \in K \quad (20)$$

$$\delta^k_i \leq y^k_i \qquad i \in C \;\; k \in K \quad (21)$$

The objective function (1) minimizes the arrival time of the truck at the depot after the last resupply is performed (i.e., the completion time). The set of routing constraints (2-7) impose that a single path serving all the customers is performed by the truck, regardless of the number of resupplies. The set of synchronization constraints (8-12) ensures consistency between the start and end of each resupply, the travel and service times, and the customer release dates. The set of resupply constraints (13-21) guarantees the feasibility of each resupply and account for the number of packages loaded onto the drone.

The proposed formulation contains an exponential number of constraints due to the subtour elimination constraints (7). Therefore, we developed a *B&C* algorithm that dynamically introduces such constraints in a lazy fashion. Moreover, the *B&C* effectiveness is further improved by the integration of the following valid inequalities:

$$\sum_{(i,j) \in Cut_{pq}} x^k_{ij} + \sum_{(j,i) \in Cut_{qp}} x^k_{ji} \geq y^k_p + y^k_q - 1 \qquad p, q \in C \;\; k \in K, \quad (22)$$

where Cut_{pq} is a generic cut on the graph G between the nodes p and q. They impose that if two nodes are served after the same resupply there should be a path connecting them. These are separated by max-flow-min-cut procedure and added at the root node of the enumeration tree.

3 PRELIMINARY RESULTS AND CONCLUSIONS

The proposed *B&C* algorithm has been tested on instances derived from Solomon (1987) using the procedure described in Archetti *et al.* (2018) for the *TSPRD*. We considered four instances (C101, C201, R101, RC101) and, for each of them, we generated *TSPRD-DR* instances with 10 and 15 customers. Then, for each customer, we generated release dates using the parameter β to tune the range of release date intervals (Archetti *et al.*, 2018). Such parameter assumes 6 different values, yielding a total of 48 instances. In all instances, the drone speed was twice the one of the truck, and the drone capacity was set to 4. Finally, the drone endurance was large enough to allow resupply at any node in the instance. We compared the *TSPRD-DR* solution with the one of the *TSPRD*. The *TSPRD* optimal solutions were obtained by solving the formulation given in Archetti *et al.* (2018). The *B&C* algorithm was coded in Python, using Gurobi 11.0 with default settings as *MILP* solver, imposing a computation time limit of one hour. The experiments were conducted on an Intel(R) Core(TM) i7-8700, 3.20 GHz, 16.00 GB RAM. Table 1 presents experimental results comparing *TSPRD-DR* with *TSPRD*. The "Savings" column shows average percentage savings from drone resupply versus truck resupply, while "Time-TR" and "Time-DR" report the average runtimes for *TSPRD* and *TSPRD-DR* formulations, respectively. Each row represents an average of four instances.

Table 1 – *Results on instances with 10 and 15 customers*

	|C| =10			|C| =15		
β	Saving	Time-TR	Time-DR	Saving	Time-TR	Time-DR
0.5	19.49	0.24	1.57	23.00	3.22	50.71
1	17.00	0.54	4.32	14.83	5.44	288.47
1.5	11.40	0.63	17.59	13.61	10.34	738.05
2	9.77	0.78	26.00	10.82	9.38	1520.70
2.5	7.50	0.86	18.18	8.06	11.93	1838.32
3	6.21	0.80	5.75	9.61	11.30	1486.66

We can observe that using the drone for resupply can lead to significant savings. Specifically, when packages become available quickly at the depot (i.e., for low values of β), larger savings are achieved. This can be attributed to the drone's superior speed compared to the truck, allowing it to swiftly reach the depot, collect newly available packages, and return to the truck efficiently. Another observation concerns the running times. The *TSPRD-DR* requires significantly longer computation times compared to the *TSP-RD*, by at least an order of magnitude. This highlights the complexity of the *TSPRD-DR*, particularly for large-size instances. Consequently, even if our formulation outperforms the one by Pina-Pardo *et al.* (2021) in the number of solved instances and computation time, future research should focus on further refining the proposed exact algorithm to scale for larger instances.

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