Time-Dependent Vehicle Routing Problem in Subway-Assisted Delivery Systems

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1 INTRODUCTION

Urban logistics is one of the most costly and time-consuming segments of goods transportation, compelling logistics companies to reduce costs and improve the efficiency of urban distribution. Nevertheless, several critical challenges persist. For instance, there are often strict restrictions on truck movements within cities, and road congestion exacerbates delays and fuel consumption, all of which increase the complexity and cost of urban logistics.

Leveraging the spare capacity of urban public transportation for logistics has emerged as a promising solution. By utilizing public transit to transfer goods from external transportation hubs to stops near their final destinations, followed by using vehicles for last-mile delivery, the overall number of vehicles required and transportation cost might be reduced. This approach not only offers significant economic benefits but also contributes to environmental sustainability by reducing greenhouse gas emissions and alleviating urban congestion. This advanced logistics model garnered interest among scholars. Readers can refer to the review by Cavallaro & Nocera (2022) for more details. A recent notable study related to this topic is Mo *et al.* (2023), which introduces the vehicle routing problem with underground logistics. They assume a large time span (e.g., the whole night) can be used to transfer the goods to the right subway stations from where the goods will be picked up by delivery vehicles. Moreover, the time-dependent travel times between point of interests are not considered.

This study focuses on the Time-Dependent Vehicle Routing Problem in Subway-Assisted delivery systems (TDVRP-SA), which simultaneously optimizes vehicle delivery routes and subway freight plans to minimize overall transportation costs. We formulate the TDVRP-SA as a mixed-integer linear programming model and develop a customized branch-and-price-and-cut algorithm to solve it efficiently. The potential contributions of this paper can be summarized as follows.

- Introduction of the TDVRP-SA that integrates urban ground transportation with subway networks to address the growing complexity of urban logistics. The TDVRP-SA considers time-dependent travel times and the collaborative use of subway capacity for freight deliveries.
- Development of a branch-price-and-cut algorithm to efficiently solve the TDVRP-SA and an innovative two-step technique employed to accelerate the algorithm.
- Execution of extensive computational experiments using extended Solomon benchmarks, demonstrating the effectiveness of the subway-assisted delivery system in reducing distribution costs and providing actionable insights for urban logistics planning.

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2 SHORT PROBLEM DESCRIPTION

In TDVRP-SA, both delivery vehicles and subway trains are involved in the process of delivering goods from the warehouse to each customer. As shown in Figure 1, vehicles depart from the depot to sequentially serve customer nodes. Throughout their routes, vehicles are replenished at either the depot or designated subway stations. Each segment of delivery between two consecutive replenishment events is defined as a *trip*, encompassing both the initial departure from and the final return to the depot as replenishment points. The travel time between each pair of points of interest is time-dependent. A complete route is thus composed of multiple consecutive trips performed by a vehicle. In the subway transport phase, goods are delivered by subway trains from the distribution center to designated subway stations, where they are temporarily stored. It is assumed that the timetable for subway trains available for freight transport is predefined.

Figure 2 provides a detailed depiction of the vehicle's space-time routes (Figure 2(a)), the temporal changes in storage levels at subway station S^* (Figure 2(b)), and the subway train timetable (Figure 2(c)). It is observed that the temporary storage at the station increases when goods are delivered by subway trains and decreases when the vehicle is replenished. Unlike a typical multi-depot vehicle routing problem, the replenishment process in TDVRP-SA necessitates coordination with the subway train timetable. Specifically, goods must be pre-delivered to the subway station by subway trains before being picked up by the vehicle, and the temporary storage at the station must not exceed its capacity limit.

In this problem, the space-time routes for vehicles to deliver goods and replenish at subway stations need to be determined, along with the decisions on which goods should be transported by which subway train to which subway station.

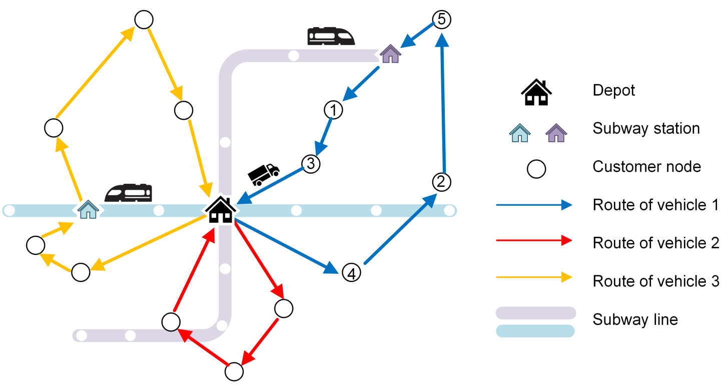


Figure 1 – *Illustration of TDVRP-SA*

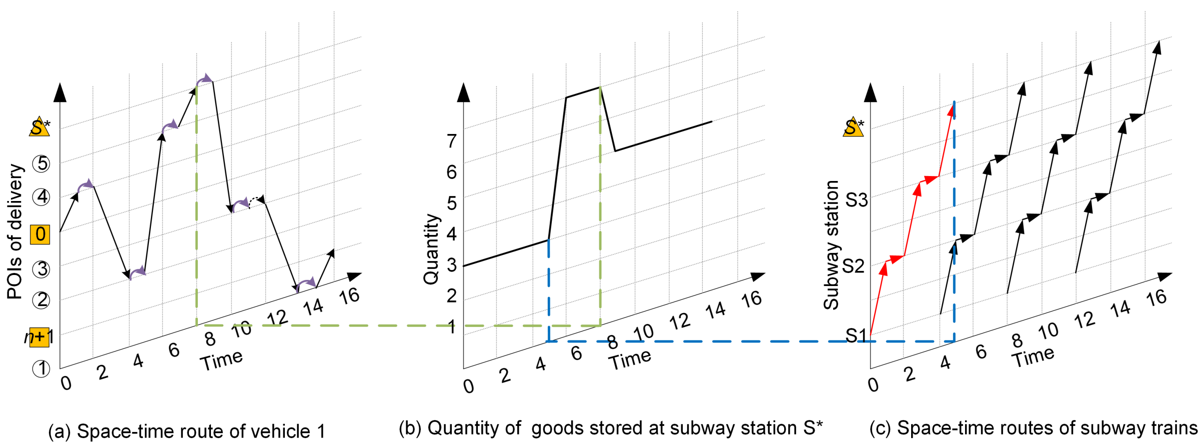


Figure 2 – *Time-Space/Time-State transitions in TDVRP-SA*

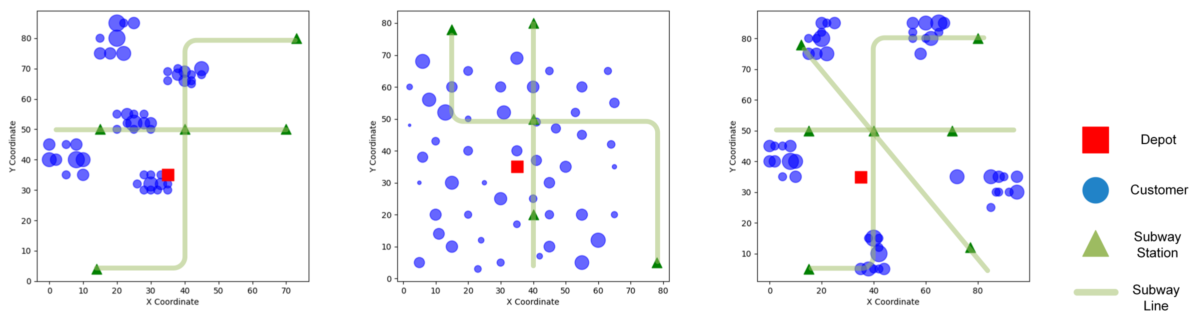
3 METHODOLOGY

TDVRP-SA is defined on a directed connected graph, denoted as $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $i \in \mathcal{N}$ represents a node and $(i, j) \in \mathcal{A}$ represents an arc. The total study period is divided into different time zones, denoted as $\tau \in \mathcal{T}$, each corresponding to a specific traffic condition for ground transportation (e.g., peak hours, off-peak hours). The vehicles are represented by $v \in V$, with each trip denoted by $m \in M$. Train service $k \in K$ is available for freight transportation.

We first developed an arc-flow model for the TDVRP-SA problem, aiming to minimize the total energy consumption of vehicles. The decision variables are $x^v_{ijm\tau}$ and y_{isk}. The 0-1 binary variable $x^v_{ijm\tau}$ equals 1 if vehicle v traverses arc (i, j) during time period τ in the m^{th} trip (0 otherwise). The binary variable y_{isk} equals 1 if the demand of customer i is transported to subway station s by train service k (0 otherwise). The model is formulated as a mixed-integer linear programming model, incorporating vehicle routing constraints, customer service constraints, replenishment constraints, and subway collaborative transportation constraints.

Due to the weak linear relaxations of the arc-flow model, we reformulated it into a set-partitioning model to achieve a stronger linear relaxation bound and developed a customized branch-price-and-cut framework to solve the partitioning model. In the set-partitioning model, the decision variables are λ_r and y_{isk}, where λ_r equals 1 if route r is selected (0 otherwise). Here, a route represents the sequence of nodes visited by a vehicle. Simultaneously solving for the variables λ_r and y_{isk} within the branch-price-and-cut framework is highly complex. Therefore, we divide the solution process into two steps: (1) solving for the vehicle routes using the branch-price-and-cut framework while ensuring the existence of a feasible solution for y_{isk}, and (2) determining the optimal solution of y_{isk} on the obtained vehicle routes.

4 RESULTS AND DISCUSSION



(a) C101-50+I (b) R101-50+II (c) RC101-50+III

Figure 3 – *Illustrations of customer type and subway station layout combinations*

To evaluate the proposed model and algorithm, experiments were conducted using the extended Solomon dataset, incorporating various subway line distributions, traffic conditions, and customer distributions to simulate realistic urban logistics and subway-assisted delivery scenarios. Three customer distribution types—Clustered (C), Random (R), and Clustered and Random (RC)—were each represented by two instances (C101, C102; R101, R102; RC101, RC102) at scales of 25 and 50 nodes. Additionally, three subway station layouts (I, II, III) were combined with these customer distributions to form complete spatial layouts, as illustrated in Figure 3(a–c) corresponding to C101-50+I, R101-50+II, and RC101-50+III. Traffic conditions were modeled by dividing the study horizon into five time periods, assigning a free-flow speed of 1.00 and a congested speed of 0.65. Two time-dependent traffic scenarios were defined: [0.65, 1.00, 1.00, 1.00, 0.65] to simulate morning and evening rush hours, and [1.00, 1.00, 1.00, 1.00, 1.00] to represent a free-flow state with no congestion.

All instances are able to reach optimal solutions within two hours, demonstrating the effectiveness of our model and algorithm. In Tables 1 and 2, we present the impact of different traffic conditions and station layouts on transportation costs under the subway-assisted mode, where "with/without SA" indicates whether the subway is involved in the delivery process. We have the following observations from results.

- The subway-assisted mode significantly reduces transportation costs, as evidenced by lower costs compared to traditional delivery methods (without SA) in all instances.
- Traffic conditions impact routing and system performance. As shown in Table 1, the subway-assisted mode reduces average transportation costs by 17.2% under conditions with congestion compared to a 15.7% reduction under free-flow conditions, indicating a more pronounced cost-saving potential in the presence of congestion.
- The placement of delivery-assisting subway stations affects the performance of subway-assisted delivery systems (see Table 2). For instance, in instance C101-25, layout 2 achieves a 26.8% greater cost reduction than layout 1. Therefore, optimizing the selection of delivery-assisting subway stations is essential for future research.

Table 1 – *Comparison of results under different Traffic conditions*

Instance	Traffic condition 1			Traffic condition 2		
	without SA	with SA	cost↓	without SA	with SA	cost↓
c101-25	68.82	61.81	10.2%	63.16	58.92	6.7%
c101-50	136.89	113.31	17.2%	126.16	107.34	14.9%
c102-25	68.79	61.13	11.1%	63.16	59.02	6.6%
c102-50	137.1	111.27	18.8%	126.56	106.3	16.0%
r101-25	72.43	67.58	6.7%	67.42	63.49	5.8%
r101-50	146.11	129.12	11.6%	135.34	119.5	11.7%
r102-25	71.98	65.18	9.4%	67.21	61.2	8.9%
r102-50	145.11	121.12	16.5%	133.57	111.23	16.7%
rc101-25	129.95	106.69	17.9%	118.81	98.99	16.7%
rc101-50	244.11	190.51	22.0%	219.26	174.82	20.3%
rc102-25	126.92	102.68	19.1%	119.01	97.94	17.7%
rc102-50	235.74	181.66	22.9%	217.39	170.04	21.8%
Average	**132.00**	**109.34**	**17.2%**	**121.42**	**102.40**	**15.7%**

Table 2 – *Comparison of results under different station layouts*

Instance	without SA	Station Layout 1		Station Layout 2		Station Layout 3	
		obj.	cost↓	obj.	cost↓	obj.	cost↓
c101-25	68.82	61.81	10.2%	43.39	37.0%	50.38	26.8%
c101-50	136.89	113.31	17.2%	107.42	21.5%	101.97	25.5%
c102-25	68.79	61.13	11.1%	42.61	38.1%	49.74	27.7%
c102-50	137.1	111.27	18.8%	104.73	23.6%	99.54	27.4%
r101-25	72.43	67.58	6.7%	69.04	4.7%	67.37	7.0%
r101-50	146.11	129.12	11.6%	134.87	7.7%	128.61	12.0%
r102-25	71.98	65.18	9.4%	64.21	10.8%	64.98	9.7%
r102-50	145.11	121.12	16.5%	127.32	12.3%	120.06	17.3%
rc101-25	129.95	106.69	17.9%	96.86	25.5%	99.46	23.5%
rc101-50	244.11	190.51	22.0%	189.29	22.5%	184.97	24.2%
rc102-25	126.92	102.68	19.1%	91.21	28.1%	92.62	27.0%
rc102-50	235.74	181.66	22.9%	177.73	24.6%	171.93	27.1%

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