A distributionally robust approach for hazmat emergency logistics with demand uncertainty and link disruption

Extended abstract submitted for presentation at the 12th Triennial Symposium on Transportation Analysis conference (TRISTAN XII)
June 22-27, 2025, Okinawa, Japan

October 7, 2024

Keywords: Distributionally robust optimization; Hazardous materials; Emergency response; Facility location and capacity

1 INTRODUCTION

Due to the harmful nature, any incidents associated with hazardous materials (hazmats) may cause tremendous threats to the surrounding people and environment during the transportation, storage, and treatment processes. When such incidences occur, a rapid emergency response can adequately mitigate the undesired impacts. The existing literature has made great effort in establishing an effective hazmat emergency logistics system. Examples include but are not limited to List (1993), Zografos & Androutsopoulos (2008), Zhao & Ke (2019), Ke *et al.* (2022). Recently, scholars have realized the prevailing uncertainties and disruptions may substantially strike an emergency system and hence should be taken into account at the system development stage, for instance, Ehsan *et al.* (2012), Ke (2022), Ke & Bookbinder (2023). In the present research, we extend the work of Ke & Bookbinder (2023) to the case with unknown distributions for both demands and link disruptions, constructing an effective hazmat emergency system by using a distributionally robust optimization (DRO) approach.

2 MODEL DEVELOPMENT

Given a road network $N(V, E)$ with vertex set V and link set E, we let EF and IS respectively to be the sets of emergency facility candidates and incident sites. For each incident site may store different types of hazmat (HM), and each emergency facility maintains multiple types of emergency units accordingly. Each pair of incident site and facility is connected by a direct link. In addition, we divide the continuous time into discrete time intervals (TM) and address the influence of the time factor by attaining the time-dependent values through detailed data analyses. The notation list is shown in Table 1.

Jointly considering the uncertain demand and link disruption, the corresponding ambiguity set can be defined as follows, where the first three constraints capture the ranges of the expected demand and disruption, the fourth shows the marginal dispersion of the random demand is around its mean, and the last one indicates the bounded supports of $\tilde{\mathbf{W}}$ and $\tilde{\delta}$.

$$\mathcal{F}_{W\delta} = \left\{ P \in \mathcal{P} \left| \begin{array}{l} \left(\tilde{\mathbf{W}}, \tilde{\delta}\right) \sim P \\ E_P\left[\tilde{\mathbf{W}}\right] \in (\mathbf{u}^-, \mathbf{u}^+) \\ E_P\left[\tilde{\delta}\right] \in (\mathbf{v}^-, \mathbf{v}^+) \\ E_P\left[\left\|(\tilde{W}_{jkt} - u_{jkt}\right\|_1\right] \leq \sigma_{jkt}, \forall j \in IS, \forall k \in HM, \forall t \in TM \\ P\left[\tilde{\mathbf{W}} \in (\mathbf{W}^-, \mathbf{W}^+), \tilde{\delta} \in \mathcal{U}_{CH}\right] = 1 \end{array} \right. \right\}. \quad (1)$$

Table 1 – *Notation*

Parameters

FC_i is the fixed cost of locating emergency facility at node i.

VC_{ik} is the variable cost of operating one emergency group for hazmat k at facility i.

AC_i is the cost of adding one unit of extra capacity on facility i.

CF_i is the capacity of facility i.

CL_{ij} is the capacity of link (i,j).

δ_{ijt} is the percentage of capacity of link (i,j) fails in time interval t.

LC_{ij} is the cost of adding one unit of extra capacity on link (i,j).

R_{ijkt} is the potential risk at incident site j caused by hazmat k in time interval t.

W_{jkt} is the emergency requirement of hazmat k at incident site j.

T_{ijt} is the response time between i and j in time interval t.

M is an infinite positive integer.

Decision variables

o_i is 1, if emergency facility is located at node i; 0, otherwise.

q_{ik} is the number of emergency groups operated by facility i for hazmat k.

f_i is the extra unit of capacity added to facility i.

e_{ij} is the extra unit of capacity added to link (i,j).

x_{ijkt} is the amount of emergency requirement of hazmat k at site j satisfied by facility i in time interval t.

Given the above-defined ambiguity set, we next present the DRO model.

$$\begin{aligned}\min_{\alpha} \sum_{i\in EF} FC_i o_i + \sum_{i\in EF}\sum_{k\in HM} VC_{ik} q_{ik} + \sum_{i\in EF} AC_i f_i + \sum_{i\in EF}\sum_{j\in IS} LC_{ij} e_{ij} \\ + \omega \sup_{p\in\mathcal{P}} E_p \left(\sum_{i\in EF}\sum_{j\in IS}\sum_{k\in HM}\sum_{t\in TM} R_{ijkt} x_{ijkt} \right)\end{aligned} \tag{2}$$

s.t.

$$\sum_{j\in IS}\sum_{t\in TM} x_{ijkt} \le q_{ik}, \qquad \forall i\in EF, \forall k\in HM; \tag{3}$$

$$\sum_{i\in EF} x_{ijkt} \ge \tilde{W}_{jkt}, \qquad \forall j\in IS, \forall k\in HM, \forall t\in TM; \tag{4}$$

$$\sum_{k\in HM} x_{ijkt} \le (CL_{ij}+e_{ij})(1-\tilde{\delta}_{ijt}) \qquad \forall i\in EF, \forall j\in IS, \forall t\in TM; \tag{5}$$

$$\sum_{k\in HM} q_{ik} \le (CF_i + f_i) o_i, \qquad \forall i\in EF; \tag{6}$$

$$f_i \le \zeta\, CF_i o_i \qquad \forall i\in EF; \tag{7}$$

$$o_i \in \{0,1\},\ f_i \ge 0, \text{integer} \qquad \forall i\in EF; \tag{8}$$

$$q_{ik} \ge 0, \text{integer}, \qquad \forall i\in EF, \forall k\in HM; \tag{9}$$

$$e_{ij} \ge 0, \text{integer}, \qquad \forall i\in EF, \forall j\in IS; \tag{10}$$

$$x_{ijkt} \ge 0, \qquad \forall i\in EF, \forall j\in IS, \forall k\in HM, \forall t\in TM. \tag{11}$$

Objective (2) minimizes the joint objective of costs for establishing the emergency logistics network (including the capital costs of constructing facilities and maintaining required emergency supplies), costs for additional capacities, and the cost-equivalent system risk (with the help of a coefficient ω applied to the risk). Constraint set (3) shows that the total amount of emergency resource can be sent out when the corresponding facility maintains enough amount. Constraint set (4) is the demand constraint, where the demand $\tilde{W}_{jkt}$ is uncertain in nature. Constraint set (5) makes sure of the link limit not being exceeded with consideration of additional capacity and disruption, where the disruption parameter $\tilde{\delta}_{ijt}$ is uncertain. Constraint set (6) gives the facility capacity restriction with extra capacity added. Finally, Constraint set (7) restricts the extended capacity can only apply to constructed facility and cannot exceed a certain portion (ζ) of the original capacity. Constraint sets (8)-(11) define the domains of decision variables.

3 SOLUTION PROCEDURE

The above DRO model is a typical location-allocation problem, which can be solved by using a decomposition-based method. Specifically, we consider the first-stage system design problem as the master problem (aiming to minimize the costs induced before the uncertainties), and the second-stage allocation problem as the subproblem (minimizing the worst expected risk resulting from the recognized uncertainties). We first reformulate the above model into a tractable formulation, and then apply a Benders decomposition approach for solutions.

The detailed transformation process is omitted here due to the space limit. Our numerical experiments show that the algorithm can solve problem instances with up to 144 nodes and 3 time intervals. The overall average CPU time for various scaled networks is approximately 1.5 hours with an average gap less than 9.5%. These results show that the Benders decomposition can provide each iteration with a shorter computational time, which allows more iterations to be completed for better gaps in the process of computation.

4 CASE STUDY

The case study is built upon the real emergency logistics management for the hazmat transportation network of Shenzhen, China, consisting of 8 emergency facility candidates and 60 incident sites.

We solve Model [DRO] and report the trade-off curve in Figure 1 with the value of ω varies between 0 and 12. Among the 13 solutions, we choose the one with $\omega = 4$ as our "intermediate" solution, which is the inflection point of the trade-off curve. The details of this solution are given in Table 2.

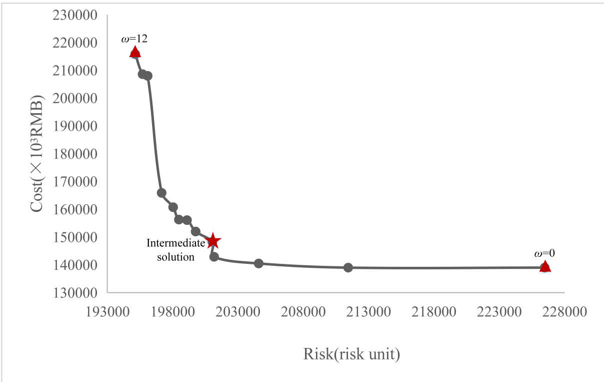

Figure 1 – *trade-off curve*

We further conduct an out-of-sample analysis to demonstrate the benefit of considering uncertainty and distributional robustness by comparing our DRO against the deterministic model (DET) and the two-stage stochastic model (TSM). Figure 2 gives the histograms of the resulting risks of the three models. DRO performs best on all out-of-sample testing metrics compared to the others, with not only the lowest risk value (respectively 12.71% and 6.78% less than DET and TSM), but also a relatively small variation among scenarios.

Table 2 – *Intermediate solution*

Cost	Facility	Add fac cap	Add link cap	Maintenance	Total	Risk	
(×10^3 RMB)	110,000	4,000	7,800	26,500	148,300	(units)	201,036
Facility details	F1 (67,26,27), F2 (89,2,86), F4 (45,115,40), F6 (43,0,120), F7 (99,71,30), F8 (78,100,22)						
Additional facility capacity	F2 (17), F6 (3)						
Additional link capacity	F1→S47 (20), F1→S62 (1), F2→S58 (5)						

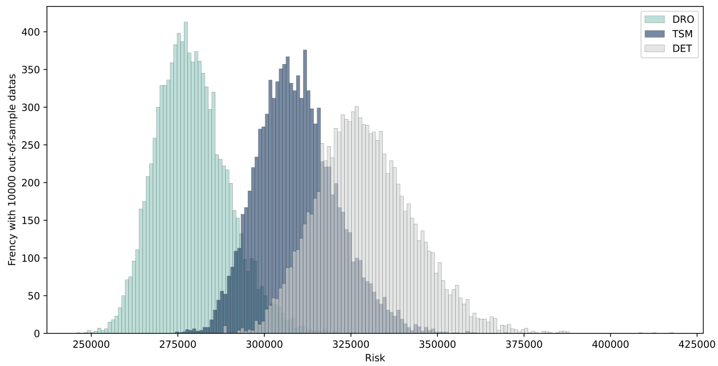

Figure 2 – *Histograms of risks of DRO, TSM, and DET for the feasible out-of-samples.*

5 CONCLUSION

This work proposes a distributionally robust approach for managing time-varying hazmat emergency logistics. The two random variables, uncertain emergency requirements and possible capacity failure of response links, are defined in a joint ambiguity set, which is then used to develop a bi-objective formulation seeking both locations of facilities and allocation of emergency services. A Benders decomposition algorithm is tailored to solve the model, which is then applied to a real-world case study for validation and insights.

References

Ehsan, E, Makui, Ahmad, & Shahanaghi, Kamran. 2012. Emergency response network design for hazardous materials transportation with uncertain demand. *International Journal of Industrial Engineering Computations*, **3**(5), 893–906.

Ke, Ginger Y. 2022. Managing reliable emergency logistics for hazardous materials: A two-stage robust optimization approach. *Computers & Operations Research*, **138**, 105557.

Ke, Ginger Y., & Bookbinder, James H. 2023. Emergency Logistics Management for Hazardous Materials with Demand Uncertainty and Link Unavailability. *Journal of Systems Science and Systems Engineering*, **32**(2), 175–205.

Ke, Ginger Y, Hu, Xun-Feng, & Xue, Xiao-Long. 2022. Using the Shapley Value to Mitigate the Emergency Rescue Risk for Hazardous Materials. *Goup Decision Negotiation*, **31**, 137–152.

List, George F. 1993. Siting emergency response teams: tradeoffs among response time, risk, risk equity and cost. *Pages 117–133 of: Transportation of Hazardous Materials*. Springer.

Zhao, Jiahong, & Ke, Ginger Y. 2019. Optimizing emergency logistics for the offsite hazardous waste management. *Journal of Systems Science and Systems Engineering*, **28**(6), 747–765.

Zografos, Konstantinos G, & Androutsopoulos, Konstantinos N. 2008. A decision support system for integrated hazardous materials routing and emergency response decisions. *Transportation Research Part C: Emerging Technologies*, **16**(6), 684–703.