Combining Lagrangian relaxation and a two-set column generation model for integrated railway freight planning

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1 INTRODUCTION

In the last decade, freight transportation has been one of the few economical sectors in Europe to experience a drastic increase in greenhouse gas emissions. This observation pushes for an increase of the share of rail freight transportation because it is one of the most energy-efficient and sustainable means of delivery. To support this increase, this work proposes a novel approach for the integrated planning of two critical resources: (1) Train paths, which are structural and temporal resources denoting parts of the network and (2) Rolling stock.

These resources are usually planned in a sequential manner, which starts from planning train paths usage, convoy composition and finally rolling stock usage. This sequential approach is used in order to reduce the problem complexity. However, it often leads to sub-optimal global solutions. We propose an approach to simultaneously plan the selection of train paths, define the convoys and schedule the rolling stock. This approach relies on a column generation scheme with two sets of columns, whose coupling constraints are relaxed to design a Lagrangian heuristic.

The effectiveness of the model is tested on real-life freight instances taken from the french network. Our solution approach is compared both to the sequential approach and to a naive Mixed Integer Linear Programming model, showing both cost reductions and savings in CPU time.

2 LITERATURE REVIEW

Simultaneous optimizing the use of multiple critical resources in railway transportation has been a field of interest since several decades, with some research already relying on Lagrangian relaxation (see e.g. Dauzère-Pérès *et al.* (2015)). In the past years, column generation has been extensively studied to tackle various decision problems in railway transportation such as timetable generation (see e.g. Pan *et al.* (2023)), classification yard planing (see e.g. Bohlin *et al.* (2012)) or rolling stock assignment (see e.g. Jaumard & Tian (2016)). The integration of train routing and

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rolling stock assignment was explored by Bach *et al.* (2015) while not considering advanced "cut and link" decisions. Pan *et al.* (2023) also study the integration of rolling stock planning and timetabling decisions in passenger transportation context with great results on industrial datasets.

Note that column generation and Lagrangian relaxation have already been hybridized to solve planning problems, notably in Huisman *et al.* (2005) for a multi-depot vehicle routing problem and in Sandhu & Klabjan (2007) for the aircraft crew-pairing problem but not for problems integrating decision levels. Also, to our knowledge, Lagrangian relaxation has never been used to solve a column generation model with two sets of columns.

Our work includes two main contributions. First, the use of a hybridized multiple column generation scheme coupled with a Lagrangian relaxation is original. Second, the simultaneous planning of train paths and rolling stock for rail freight transportation has rarely been studied on congested European railway networks with "passenger first" policies.

3 PROBLEM DEFINITION AND MODELING

Since the integrated planning of train routing and rolling stock assignment is tackled in this paper, let us briefly present the two individual problems in Sections 3.1 and 3.2, respectively. Then, the first column generation heuristic with two sets of columns is presented in Section 3.3, and the Lagrangian relaxation heuristic of the column generation model in Section 4.

3.1 The convoy planning problem

In the convoy planning problem, both the selection of a subset of train paths that will be used for the weekly transportation plan and the composition of convoys are optimized.

Each freight demand specifies the locations for both its origin and its destination, as well as the required service times (an availability time window for pickup and an availability time window for delivery). Carriages can be combined into massive convoys at specific points of the network (classification yards). This combination of carriages at rail yards, named "cut and link" operation, is critical for an optimal usage of resources and enforces a strict process time. A feasible route for a demand will be referred to as a **flow route**.

3.2 The rolling stock planning problem

Weekly trips are defined for each rolling stock unit. Each trip must start and end at specific locations marked as “depots”. A rolling stock unit can circulate using four modes: (1) *Single traction*, the rolling stock unit is simply assigned to tract a convoy, (2) *Double traction*, the rolling stock unit is paired with another engine, effectively doubling its traction power, (3) *Deadheading*, the rolling stock unit is not pulling a train and circulates alone for repositioning, and (4) *Vehicle repositioning*, the rolling stock unit is moved as part of a larger convoy, being pulled by other rolling stocks.

Rolling stock units can be coupled at any point of the network, but specific chaining time must be observed depending on structural constraints. A feasible route for a rolling stock unit is referred to as a **rolling stock route**.

3.3 Modeling

Available train paths are modeled using a space-time connection graph in such a way that feasible routes in the graph correspond to feasible flow and rolling stock routes. Feasible flow routes are computed using a Dijkstra-like approach.

We leverage a two-set column generation scheme where $\mathcal{T} = \bigcup_{f\in\mathcal{F}} \mathcal{T}_f$ is the set of *feasible flow routes* and $\mathcal{C} = \bigcup_{l\in\mathcal{L}} \mathcal{C}_l$, the set of *feasible rolling stock routes*. In the simplified model (Equations (1) and (2)), α_t is the gain associated to satisfying flow route t, c_t and c_c are the costs of flow route t and rolling stock route c respectively, $\beta_{t,p}$ is the induced weight of flow route t on train path p and $\gamma_{c,p}$ is the traction force of rolling stock route c on train path p. Objective function (1) selects the best coupling of column selection variables (Z_t for flow routes and Y_c for rolling stock routes) under a weight coupling constraint (2).

The set of flow routes $\mathcal{T}$ is pre-computed exhaustively, while a column generation scheme is employed to generate the best rolling stock routes $\mathcal{C}' \subset \mathcal{C}$.

Simplified model:

$$\max \sum_t Z_t(\alpha_t - c_t) - \sum_c Y_c c_c \quad (1)$$

s.c.

[Operational constraints]

$$\forall p \in \mathcal{P} \qquad \sum_t Z_t \beta_{t,p} - \sum_c Y_c \gamma_{c,p} \leq 0 \quad (2)$$

4 A LAGRANGIAN RELAXATION HEURISTIC

The two sets of columns considered in our approach make it difficult to use a branch & price approach to solve the problem exactly. While rounding heuristics gave promising results on small instances, the integrality gap is much larger on industrial instances and the computational times were deemed too large for practical uses.

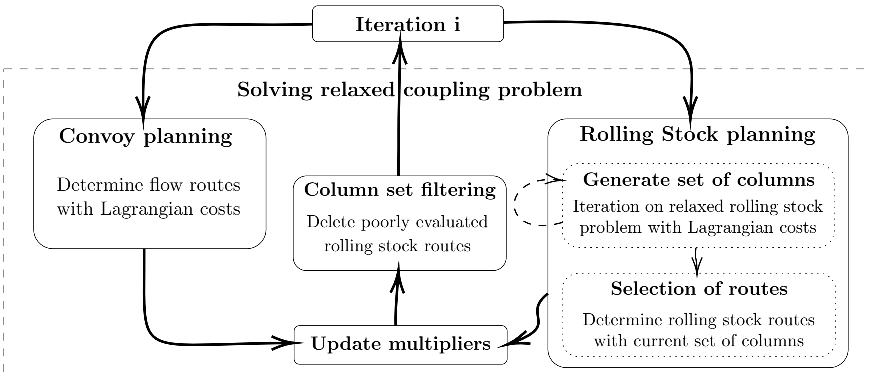

Figure 1 – *Resolution scheme of the Lagrangian relaxed two-set column generation model.*

We propose a Lagrangian relaxation of our column generation primal problem by penalizing the coupling constraints (2) using Lagrangian multipliers μ_p, leading to the two sub-problems below:

$$\max \sum_t (\alpha_t - c_t - \sum_p \mu_p \beta_{t,p}) Z_t \quad (3)$$

s.c.

[Flow route constraints]

$$\max - \sum_c (c_c - \sum_p \mu_p \gamma_{c,p}) Y_p \quad (4)$$

s.c.

[Rolling stock route constraints]

Relying on the fact that the generation of columns for sub-problems (3) and (4) can be performed relatively fast, each master iteration of the Lagrangian heuristic consists of solving to optimality the relaxed sub-problems and solving the resulting model by a standard solver, as illustrated in Figure 1. A standard sub-gradient algorithm is used to converge to the Lagrangian dual.

5 NUMERICAL RESULTS

The comparison of freight indicators obtained by the integrated approach and the standard sequential approach confirmed that consistent gains can be achieved by simultaneous planning the rolling stock and selecting the train paths. The first results on our Lagrangian heuristic seem to confirm its advantages over the direct column generation approach.

	CPU Time (s.)		Flow satisfaction		Rolling Stock units	
	CG	CGL	CG	CGL	CG	CGL
Small	13	8	109	109	11	9
Medium	59	29	169	169	38	39
Large	1782	876	1319	1319	72	72
Gen-30	52	18	26	25	4	4
Gen-40	43	28	34	35	5	5
Gen-50	42	38	41	43	6	6
Gen-80	121	112	67	64	7	6

Table 1 – *Comparison of solution indicators for the direct two-set column generation approach (CG) and the Lagrangian heuristic of the two-set column generation model (CGL)*

Table 1 shows the results for the first two-set column generation approach (see Section 3.3) and the Lagrangian heuristic of the two-set column generation model (see Section 4). Column "Solution Time" gives the computational time for each approach, Column "Flow satisfaction" indicates the number of freight demands that are satisfied, and Column "Rolling Stock units" gives the number of rolling stock units needed to cover the created convoys. CGL performs significantly faster than the CG approach, with a computational time saving of around 50% on large instances. The number of covered demands is the same for both approaches. CGL also performs better on the small instance, covering demands with only 9 rolling stocks. This is likely due to the optimality loss induced by the integer resolution heuristic of the direct column generation approach.

Our models were also tested on a set of randomly generated instances on the French network. For those instances, the dominance of CGL compared to CG in terms of solution quality is not so clear, but the computational time is always smaller with CGL. Besides the computational times, the results on some larger instances do not show improvements with integrated planning over sequential planning. These results are probably related to the efficiency of the sub-gradient algorithm in the Lagrangian heuristic, and we are confident that it can be improved.

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