An exponentiated random utility model (ERUM): Properties and application to bounded travel choice

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1 INTRODUCTION

In the era of emerging technologies, the rapidly developing information technologies offer travelers increasing access to advanced traffic information. For instance, travelers can easily obtain suggested trip plans from the mapping software and understand the possible range of travel costs thereof. Based on the lower and upper bounds of travel costs, travelers can filter out a set of satisfactory travel alternatives and are likely to select alternatives close to the lower cost bound while avoiding alternatives close to the upper cost bound. Thus, the provision of advanced traveler information may not only enhance transportation efficiency, but also significantly reshape the disaggregate travel behavior. In future transportation systems with adequate traffic information, accurately modeling the individual behavioral responses to advanced traveler information is an imperative task for analyzing the travel demand pattern and evaluating transportation plans and policies.

Random utility maximization models (RUMs), such as the traditional logit-based additive RUM (ARUM, Domencich and McFadden, 1975) and the recently developed weibit-based multiplicative RUM (MRUM, Fosgerau and Bierlaire, 2009; Gu et al., 2022), play a dominant role in travel choice modeling. Benefiting from the properties of the Gumbel and Weibull error distributions, logit and weibit models retain the functional form of the Luce (1959) model's probability expression. The "Luce-form" choice probability (as termed by Mattsson et al., 2014) enables easy evaluation of travel choice and facilitates the estimation and interpretation of logit and weibit models. However, the unbounded Gumbel and Weibull error distributions imply unbounded perceived travel utility, making the logit and weibit models inherently difficult to reflect the effect of lower and upper cost bounds in travel perception and decision-making.

This behavioral issue can be partly addressed by incorporating a choice set formation stage in the choice modeling (Manski, 1977), where alternatives with high attractiveness can be included in the choice set for further consideration in the choice-making stage. Besides determining the choice set in an exogeneous stage, several choice models have been developed to endogenously exclude unattractive alternatives in network equilibrium analysis, including the bounded choice model (BCM, Watling et al., 2018) and truncated choice model (TCM, Tan et al., 2024). By truncating the logit model formulation, BCM and TCM assign zero probabilities to alternatives outside the upper cost bounds determined based on equilibrium route costs. Hence, the subset of alternatives within the lower and upper costs bounds can be endogenously considered with choice probability obtained by the modified logit probability function.

Besides focusing on the role of cost bound in choice set formation, the behavioral effect of cost bound in travelers' decision-making has not received enough attention. Firstly, the logit-based choice-making behavior within the specified cost bounds is dependent on the absolute difference between travel costs, which is difficult to capture the relative advantages of different alternatives within the cost bounds (Gu et al., 2022). Also, the explicit information of cost bounds may also influence how travelers perceive the subset of alternatives within cost bounds. Furthermore, the truncation in logit-based probability expression lacks a solid behavioral interpretation consistent with the random utility theory underlying the logit model.

This study aims to propose an advanced RUM approach to tackle the abovementioned behavioral issues. A novel exponentiated utility function is considered to develop the exponentiated random utility model (ERUM). The doubly bounded Kumaraswamy distribution is adapted to model travel perception errors. Benefiting from the novel utility functional form and error distributional assumption, the proposed model is capable of considering the effects of both lower and upper cost bounds while retaining a tractable closed-form probability expression. In the remaining abstract, the formulation of the proposed model is first presented in Section 2, followed by a brief illustration of the model properties in Section 3. Finally, the concluding remarks are presented in Section 4.

2 MODEL FORMULATION

Instead of the additive or multiplicative form of utility function assumed in the ARUM and MRUM frameworks, inspired by the exponentiated transformation widely used in the reliability engineering, the ERUM framework considers an exponentiated functional relationship between the deterministic utility and random error term, which is

$$U_k = \left(\varepsilon_k\right)^{u_k}, \tag{1}$$

where u_k and ε_k denote the deterministic utility and random error terms, respectively.

Compared with the absolute/relative utility difference implied by the additive/multiplicative utility function used in ARUM/MRUM, the exponentiated utility function provides a novel alternative to depict how individuals perceive and compare different magnitudes of travel costs. Taking the utility maximization problem as an example, the choice probability of alternative *k* is the probability that *k* has higher travel utility than all other alternatives in choice set *K*, i.e.,

$$\begin{aligned} P_k &= \Pr\left\{U_k \geq U_i, \forall i \neq k \in K\right\}, \forall k \in K \\ &= \Pr\left\{\varepsilon_i \leq \left(\varepsilon_k\right)^{u_k/u_i}, \forall i \neq k \in K\right\} \\ &= \int_{-\infty}^{\infty} F_k\left(\left(\varepsilon_k\right)^{u_k/u_1}, \ldots, \left(\varepsilon_k\right)^{u_k/u_{|K|}}\right) d\varepsilon_k \end{aligned}, \tag{2}$$

where $F_k(.)$ denotes the partial derivative of CDF of ε with respect to ε_k.

The Kumaraswamy distribution has been obtained and applied to describe random variables bounded on both sides (Kumaraswamy, 1980). The probability density function (PDF) and cumulative distribution function (CDF) of the Kumaraswamy distribution are

$$f\left(x\right) = abx^{a-1}\left(1-x^a\right)^{b-1}, 0 < x < 1, \tag{3}$$

$$F\left(x\right) = 1-\left(1-x^a\right)^b, 0 < x < 1, \tag{4}$$

where *a* and *b* are two positive shape parameters. To retain the Luce-form probability expression, we consider the Kumaraswamy distribution with a unit shape parameter *b*=1. The choice probability of Kumaraswamy-based ERUM can then be obtained by substituting Eqs. (3) and (4) into Eq. (2):

$$P_k = \frac{\left(u_k\right)^{-1}}{\sum_{i \in K}\left(u_i\right)^{-1}}. \tag{5}$$

u_k can be represented via the expected utility m_k, $u_k = (1-m_k)/m_k$. As m_k ranges from 0 to 1, the travel cost c_k should be standardized to convert to the expected utility, i.e., $m_k = \frac{g_K - c_k}{g_K - l_K}$ (Kumaraswamy, 1980), where g_K and l_K denote the upper and lower cost bound, respectively. The choice probability can then be written as follows:

$$P_k = \frac{\dfrac{g_K - c_k}{c_k - l_K}}{\sum_{i \in K} \dfrac{g_K - c_i}{c_i - l_K}}. \tag{6}$$

3 Model properties

Taking advantage of the exponentiated utility function and the novel distributional assumption, the proposed model possesses appealing properties for capturing various behavioral effects of the lower and upper travel cost bounds. In Section 3.1, we first illustrate the effects of cost bounds on travel perceptions based on the alternative-specific perception variance dependent on both travel utility and cost bounds. The effect on choice-making is then demonstrated in Section 3.2, where the choice probabilities are shown to be influenced by not only the costs of alternatives themselves, but also the relative location of travel alternatives within the lower and upper cost bounds.

3.1 Perception variances

Based on the mean and variance of the Kumaraswamy distribution with a unit parameter b=1, the perception variance ${\sigma_k}^2$ of the proposed model can be expressed dependent on the expected utility m_k as follows:

$$ {\sigma_k}^2 = \frac{m_k \left(1 - m_k\right)^2}{2 - m_k}, 0 < m < 1 \tag{7}$$

Figure 2 shows the relationship between perception variance and expected utility of the proposed Kumaraswamy-based ERUM. Unlike the perception variances of existing RUMs which are either constant (e.g., logit model) or proportional to the square of expected disutility (e.g., weibit model), the proposed model has a parabolic relationship between variance and expected utility – the variance eliminates when the expected utility approaches both ends and reaches the peak near (not exactly in) the middle of the cost range. With the lower and upper bounds as explicit references of travel cost, the superiority of alternatives close to the lower cost bound and the inferiority of alternatives close to the upper cost bound tend to be perceived more clearly by travelers. This appealing property implies that both the cost of travel alternative and locations of lower/upper cost bounds are endogenously considered in the travel perception and hence in the modeling of utility maximization choice behavior. Thus, the proposed Kumaraswamy-based ERUM may be suitable for choice contexts where individuals have good knowledge of the lower and upper bounds of utility, e.g., the future transportation system equipped with the advanced traveler information system and connected vehicles.

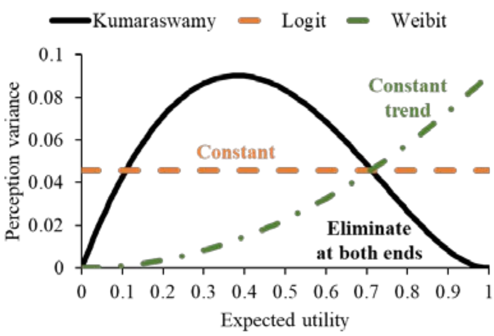
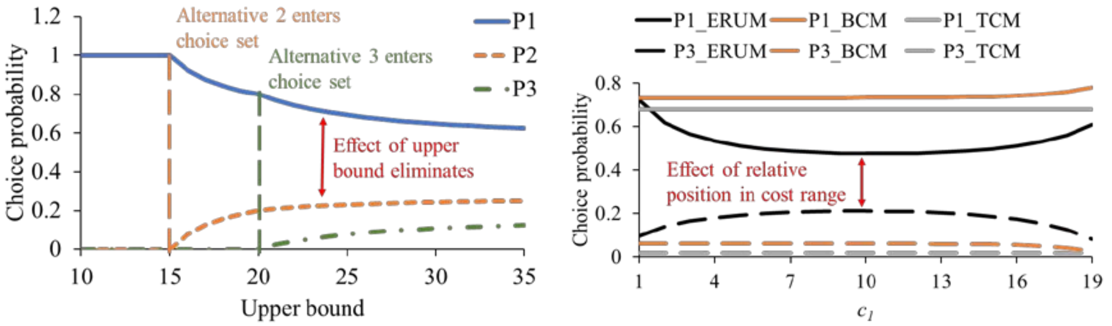


Figure 2. Utility-dependent perception variance of the Kumaraswamy-based ERUM

3.2 Effects of considering cost bounds

This section illustrates the effects of considering lower and upper bounds in the proposed model via a trinomial choice context with constant cost difference. Figure 3(a) examine the case with fixed travel costs c_1 = 5, c_2 = 10, c_3 = 15 and varying upper bounds. The relaxation of upper bounds tends to include more alternatives into consideration. The impact of upper bound on choice probabilities eliminate when the upper bound moves away the actual travel costs. Figure 3(b) considers the case with fixed lower and upper bounds ($0 and $25) and varying travel costs c_1, $c_2 = c_1$+2.5, $c_3 = c_1$+5. Compared with BCM and TCM that have relatively unchanged outcomes with respect to the cost variation, the proposed model effectively captures the relative location of travel cost within the lower and upper bounds. This indicates the capability of the proposed model to reflect how the travel cost bounds affect not only choice set formation but also travel perception and decision-making.



(a) (b)

Figure 3. Effects of (a) upper bound and (b) upper bounds in the Kumaraswamy-based ERUM

4 CONCLUSIONS

This abstract proposes a novel Kumaraswamy-based ERUM, an advanced RUM with an exponentiated utility function and doubly bounded distributional assumption. On this basis, the proposed model can effectively consider the impact of travel cost bounds while retaining the solid behavioral interpretation of the random utility theory and the tractability of Luce-form probability expression. Future efforts will be made to investigate more theoretical properties of the proposed model and develop an analytical estimation approach.

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