Discovering and Quantifying Extreme Failure Scenarios through Graph Learning for Road Transportation Systems

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1. INTRODUCTION

Evaluating the resilience of engineering systems under extreme events is essential for implementing measures to mitigate potential risks and reduce losses in life and property[1,2]. Due to the rarity of extreme events and the significant computational burden associated with system performance evaluation, estimating the probability of extreme failures can be prohibitively expensive. Importance sampling can enhance the sampling efficiency for extreme failure scenarios where system performance deteriorates markedly. However, its computational complexity escalates rapidly with the scale of the system and the dimension of variables[3,4], which limits its applicability in large-scale engineering systems. In this work, we propose a graph learning approach that successfully reduces the cost of importance sampling in large-scale infrastructure networks, exemplified by transportation networks.

2. METHODOLOGY

The proposed approach, Importance Sampling based on Graph Auto-Encoder (GAE-IS), consists of three main components: pre-sampling on a sub-network using the Cross-Entropy (CE) method[5] (Fig. 1**d**), identification of key components through GAE (Fig. 1**e**), and extreme failure probability estimation (Fig. 1**f**). In this methodology, the crude Monte Carlo simulation is utilized to randomly sample network failure scenarios on the sub-network, assuming a homogeneous link failure probability. This process facilitates the identification of critical links for maintaining network functionality—specifically, those whose failure results in substantial performance degradation, given that all links share the same failure probability. The sampled failure scenarios are then ranked in descending order of their consequent network performance indicator, Average Travel Time (ATT). A predetermined percentage (i.e. the ρ percentile) of these samples, which show significantly deteriorated performance, are selected as risk scenarios. The threshold of ATT for filtering out these risk scenarios is denoted as θ_r.

The likelihood of each link being included in failed link sets under risk scenarios reflects its functional criticality; a higher likelihood indicates greater criticality. This likelihood is employed to adjust the failure probability of each link, increasing it for critical links and decreasing it for less critical ones, thereby facilitating importance sampling. New samples are then generated based on the adjusted failure probabilities. Subsequently, the risk scenarios are updated using a new risk scenario threshold, θ_r', determined as the ρ percentile of the latest sample set. This iterative process continues until all risk scenarios are classified as extreme failure scenarios, wherein system performance falls below a specified threshold θ_e (θ_e is the value at the percentile that is rarer than θ_r in the tail of the ATT distribution). The pre-sampling process yields the likelihood of sub-network links, which serve as link labels for training the Criticality Identifier. This Criticality Identifier, based on the GAE model, uses both the network's topology and its supply-demand characteristics as inputs. The transferability of the model allows it to be trained on the sub-network and then providing importance weights for components in the larger original network. Following this, failure scenarios for the larger-scale target network are sampled based on these estimated likelihoods and homogeneous or heterogeneous structural failure probabilities of links. The probability of extreme failure scenarios is then estimated as follows:

$$\hat{p}_q = \frac{1}{n}\sum_{i=1}^{n} \frac{\mathrm{I}_{\boldsymbol{x}_i} g(\boldsymbol{x}_i)}{q(\boldsymbol{x}_i)}, \ \boldsymbol{x}_i \sim q \tag{1}$$

where $\hat{p}_q$ is the estimated probability; n is the sample size; $\boldsymbol{x}$ represents independent variables, denoting a set of failed links in a specific failure scenario; the indicator function $\mathrm{I}_{\boldsymbol{x}_i}$ takes the value 1 if the failure scenario is an extreme failure scenario, and 0 otherwise; $g(\boldsymbol{x}_i)$ is the probability density function of a failure scenario determined by the structural failure probability of each link, computed as Eq. (2), and $q(\boldsymbol{x}_i)$ is the Importance Sampling Density (ISD) function determined by the likelihood and structural failure probability of each link, computed as Eq. (3). $w = g(\boldsymbol{x})/q(\boldsymbol{x})$ is the importance weight for the failure scenario with the set $\boldsymbol{x}$ failed.

$$g(\boldsymbol{x}) = \prod_{i=1}^{N_f} \tau_{l_i} \prod_{j=1}^{N-N_f} (1-\tau_{l_j}), \quad l_i \in \boldsymbol{x},\ l_j \notin \boldsymbol{x} \tag{2}$$

$$q(\boldsymbol{x}) = \prod_{i=1}^{N_f} \varphi(s_{l_i}\tau_{l_i}, 1) \prod_{j=1}^{N-N_f} (1-s_{l_j}\tau_{l_j}), \quad l_i \in \boldsymbol{x},\ l_j \notin \boldsymbol{x} \tag{3}$$

$$\varphi\,(s_{l_i}\tau_{l_i}, 1) = \min\,(s_{l_i}\tau_{l_i}, 1) \tag{4}$$

where N_f is the number of failed links in the failure scenario with the set $\boldsymbol{x}$ failed, τ_{l_i} ($\tau_{l_i} > 0$) is the structural failure probability of link l_i, N is the number of links in the network. The modified likelihood obtained by the Criticality Identifier is denoted as s_{l_i} ($s_{l_i} > 0$), computed as Eq. (5).

$$s_{l_i} = \begin{cases} \frac{2\eta N \hat{h}_{l_i}}{\sum_{l_i \in B} \hat{h}_{l_i}} & l_i \in B \\ 1 & l_i \notin B \end{cases} \tag{5}$$

where $\hat{h}_{l_i}$ is the estimated likelihood output by Criticality Identifier. The $\hat{h}_{l_i}$ of all links is sorted in descending order. The top η proportion of links are critical links, and the bottom η proportion of links are non-critical links. B is the set of critical and non-critical links.

The proposed methodology has the potential for application across various infrastructure systems characterized by network topological features and cyber or physical flows transferring within it, including water distribution, power, and communication networks. Furthermore, GAE-IS effectively decouples component functional criticality from the risk of failure, with the latter being associated with the structural fragility of components and the spatial distribution of disruptive intensity caused by potential hazards, collectively referred to as the vulnerability distribution. (Fig. 1**b**). This means that even if the spatial distribution of component vulnerability changes, there is no need to reassess the functional criticality of components, enabling the rapid derivation of ISD for large-scale networks under multiple hazard scenarios. The transferability of Criticality Identifier and decoupling features of GAE-IS significantly reduce sampling costs in large-scale networks.

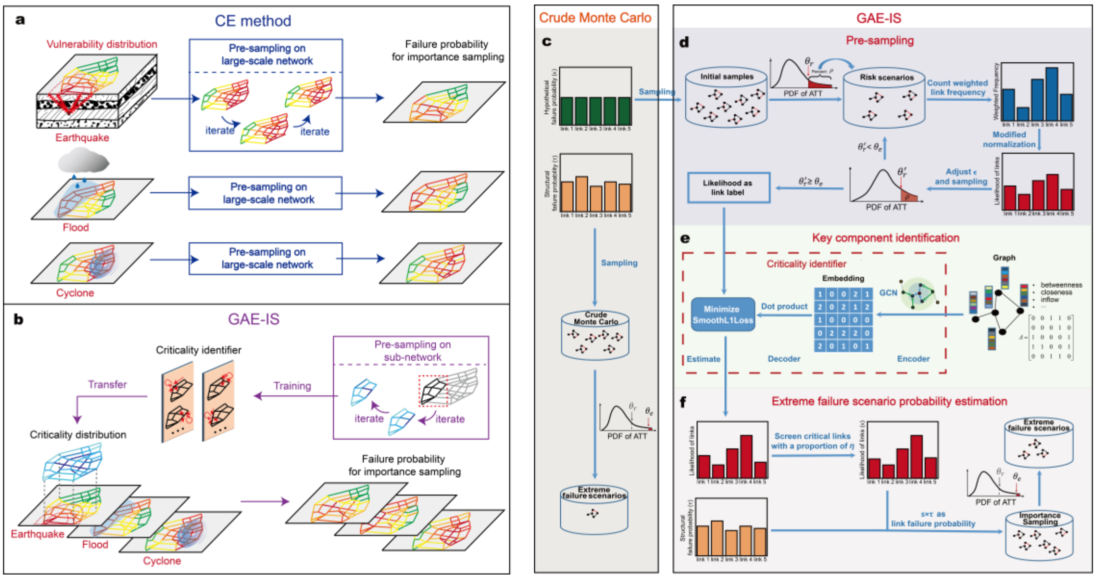

Fig. 1 Comparison of GAE-IS and the CE method, along with an overview of the GAE-IS workflow for estimating extreme failure probabilities in road networks. **a** Schematic diagram of the CE method. **b** Schematic diagram of the GAE-IS method. **c** Crude Monte Carlo method employed for pre-sampling and serving as a baseline for sampling extreme failure scenarios. **d** Pre-sampling process in GAE-IS to estimate the likelihood of each link being included in failed link sets under risk scenarios. **e** Identification of key components and estimation of link likelihood using the Criticality Identifier. **f** Sampling extreme failure scenarios based on the link likelihoods provided by the Criticality Identifier.

3. RESULTS

We evaluated the performance of the proposed GAE-IS method in road transportation systems subjected to multiple link failures. Experimental results on road networks in Berlin, Anaheim, the Northern Gold Coast under homogeneous failure risks, and Chicago under heterogeneous seismic risks demonstrate that, compared to crude Monte Carlo simulations, the proposed method captures more extreme failure scenarios with the same sample size (Fig. 2), improving sampling efficiency by 1-2 orders of magnitude and providing more accurate probability estimates. Our findings also indicate that the sampling results are sensitive to the proportion of critical links screened within the target network. To further elucidate the factors contributing significantly to the functional criticality of links, we conducted perturbation experiments[6] on the node features. Given that the Criticality Identifier comprises two graph neural network layers, its outcomes are influenced solely by the 2-hop neighbors of each node. Consequently, we perturb the node features within the 2-hop range of each node by substituting them with the mean value of that feature across all nodes, and subsequently monitor the magnitude of changes in the estimated link likelihood. The results demonstrate that the remaining capacity of nodes is the most significant feature across all networks. Although its impact varies among different road networks, this finding motivates us to further explore the potential of the GAE-IS method for cross-network transferability.

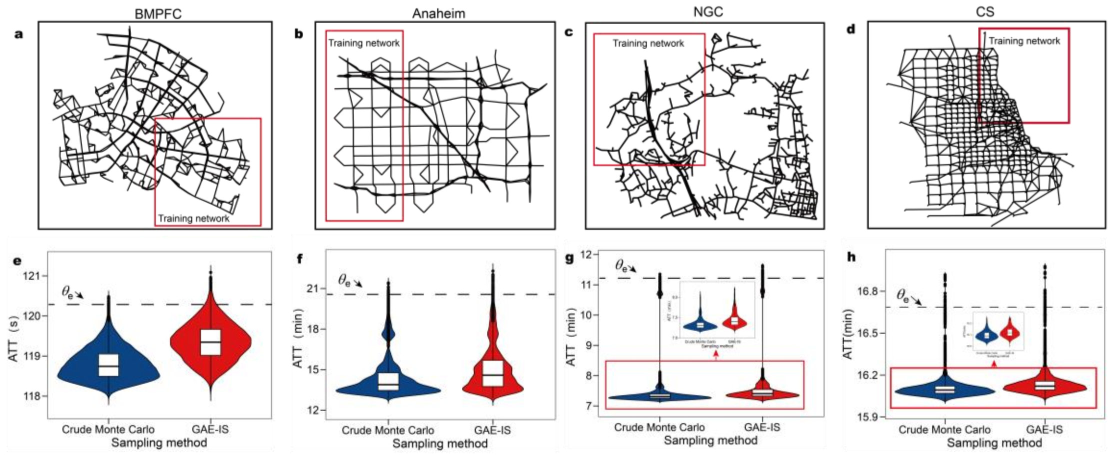

Fig. 2 Distribution of ATT for failure scenarios sampled for road networks in Berlin (BMPFC), Anaheim, Northern Gold Coast (NGC) and Chicago (CS).

4. CONCLUSIONS

The GAE-IS framework proves to be an effective and efficient tool for evaluating the resilience of infrastructure systems from the perspective of extreme value statistics. It significantly reduces the computational costs associated with sampling extreme failure scenarios that can lead to substantial performance degradation in network-structured infrastructure systems. Central to our methodology is the development of the Criticality Identifier—a graph learning model that combines topological centrality metrics with flow attributes of network components to assess link criticality. The decoupling of link criticality from failure risk allows for the integration of link criticality distribution with various vulnerability distributions, thus facilitating a more efficient acquisition of ISD functions. This methodology is particularly applicable for systems that require substantial computational effort for performance evaluation, especially in the context of analyzing extreme scenarios. The advantages of GAE-IS suggest promising applications in several key areas, including the design and operation of resilient infrastructure systems, the development of resilient cities, and the advancement of sustainable communities.

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