Urban Air Mobility Service Network Design: Ridership Maximization and Exact Solution Algorithm

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1 Introduction

In the past decades, passengers' travel patterns have been reshaped by the new modes of mobility services, one of which is the urban air mobility (UAM). UAM utilizes vertical airspace for urban and regional transportation, offering flexible and efficient transport services by leveraging aviation and autonomous technologies. Companies like Volocopter and Airbus are actively developing air taxis. Certainly, the widespread adoption of UAM requires designated skyports and air traffic management systems. Efficiently placing skyports is thus crucial. This study proposes a modeling framework for skyport locations, aiming to maximize the ridership.

Several studies have explored the strategic planing of UAM. For example, Holden and Goel (2016) utilized k-means clustering to optimize skyport locations in Los Angeles and London, focusing on long-distance Uber routes. Rajendran and Zack (2019) estimated demand for air taxis and recommended skyport locations using a two-phase approach: first, identifying potential users, and second, applying constrained clustering based on projected demand using New York City taxi data. Willey and Salmon (2021) modeled the problem as a modified single-allocation p-hub median problem, proposing five heuristic algorithms to generate feasible solutions. Rath and Chow (2022) integrated user mode choice behavior into their p-hub median model that maximizes ridership and revenue. Chen et al. (2022) addressed skyport selection with the objective of minimizing travel costs while adhering to area constraints. Recently, Kitthamkesorn and Chen (2024) developed a mathematical programming model for skyport placement, using the "eUnit" discrete choice model to account for travelers' mode choices and travel costs.

In this paper, we study the hub location problem of a "two-station" UAM system. Unlike the location models mentioned above, our model jointly optimizes the skyport locations and the service route design, i.e., we also consider how to deploy the UAM link given the skyport locations. We focus on the modal split between selecting the UAM service and the ground-only-transport option. We then propose a mixed-integer convex program to maximize the ridership. Moreover, we propose a branch-and-cut Benders decomposition algorithm. Our computational experiment demonstrates the efficiency of the proposed model and algorithm.

2 Problem description

We study the location problem of the UAM skyports (set I), aiming to maximize the ridership of the UAM service. We assume passengers are aggregated at geographic demand zones (set M), and they are traveling from one zone to the other, creating the demand for each Origin-and-Destination (OD) pair. We use $d_{mn}, \forall m \in M_n, n \in M$, to denote the demand of OD-pair (m, n). Here $M_n \subseteq M$ represents the set of origins that have "nontrivial" demand to the destination n.

As shown in Figure 1, passengers can either use the "ground-only" path $(m \rightarrow n)$, which directly connects these two zones by private vehicles with a traveling time t_{mn}; or they can choose the multimodal options that use UAM in part of their trips. For example, they can assess station i by ground-transport, take the UAM to station j, and reach the destination by ground-transport. This process generates path $m \to i \to j \to n$ whose travel time is $\tilde{t}_{mijn} = t_{mi}+t_{ij}+t_{jn}$. If a link is not selected by the operator to provide the service, then passengers cannot use it.

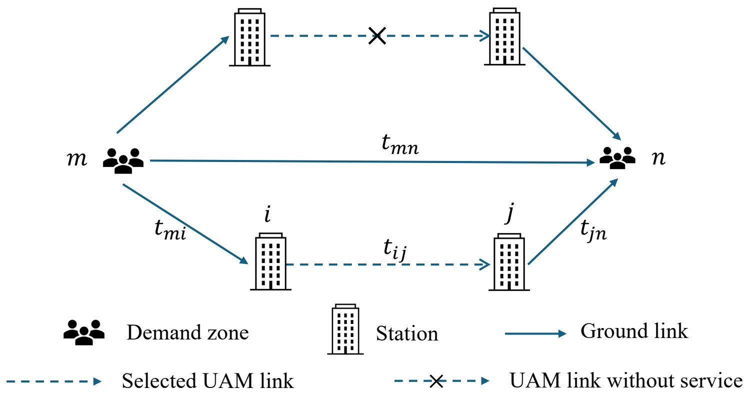


Figure 1 – *Ground-only path and multimodal paths with selected UAM links.*

To forecast the ridership, we assume that a passenger selects a path based on the random utility theory. In our setting, the utility of the path $m \to n$ is modeled as

$$U_{mn} = u(t_{mn}) + \epsilon_{mn}, \forall (m,n) \in K \tag{1}$$

where $u(t_{mn})$ is the observable utility, defined as a decreasing function of t_{mn}. ϵ_{mn} is the unobservable utility that is assumed to be a random term following some distribution. Similarly, the utility of the multimodal path $m \to i \to j \to n$ is modeled as

$$\tilde{U}_{mijn} = \tilde{u}(\tilde{t}_{mijn}, t_{ij}) + \epsilon_{mijn}, \forall i,j \in I, (m,n) \in K \tag{2}$$

where $\tilde{u}(\tilde{t}_{mijn}, t_{ij})$ is the observable utility, defined as a decreasing function of $\tilde{t}_{mijn}$ and t_{ij}.

We use the binomial logit model (BLM) where the *consideration set* consists of two options, i.e., the ground-only path and the multimodal path with the maximal observable utility. Let $x_i, \forall i \in I$ be a binary variable, which is 1 if skyport i is open. Skyport i has a fixed cost of f_i. Define $r_{ij}, \forall i \in I, j \in I$ as a binary variable, which is 1 if the service at link $i \to j$ is provided. The fixed cost of activating service is c_{ij}. Then, let y_{mijn} be a binary variable such that $y_{mijn} = 1$ if path $m \to i \to j \to n$ is in the consideration set of OD pair (m,n). We propose the following Service Network Design Problem for UAM system (SNDP):

$$\max \sum_{(m,n)\in K} d_{mn} \frac{\sum_{i\in I}\sum_{j\in I} e^{\tilde{u}(\tilde{t}_{mijn},t_{ij})} y_{mijn}}{\sum_{i\in I}\sum_{j\in I} e^{\tilde{u}(\tilde{t}_{mijn},t_{ij})} y_{mijn} + e^{u(t_{mn})}} \tag{3}$$

$$\text{st.} \sum_{i\in I} f_i x_i + \sum_{i\in I}\sum_{j\in I} c_{ij} r_{ij} \le B \tag{4}$$

[SNDP]

$$r_{ij} \le x_i, \quad r_{ij} \le x_j, \quad \forall i,j \in I \tag{5}$$

$$r_{ii} = 0, \quad \forall i \in I \tag{6}$$

$$r_{ij} = 0, \quad \text{if } L_{ij} > L^*, \forall i \in I, j \in I \tag{7}$$

$$\sum_{i\in I}\sum_{j\in I} y_{mijn} = 1, \quad \forall (m,n) \in K \tag{8}$$

$$y_{mijn} \le r_{ij}, \quad \forall i,j \in I, (m,n) \in K \tag{9}$$

$$y_{mijn} \ge 0, \quad \forall i,j \in I, (m,n) \in K \tag{10}$$

$$x_i \in \{0,1\}, \quad \forall i \in I \tag{11}$$

where the objective is to maximize the expected number of users. Constraint (4) is the budget constraint, limiting the costs of locating skyports and activating the UAM service links to B. Constraint (5) and (6) enforce that the UAM service at link $i \to j$ can be provided only if both station i and station j are open; and link $i \to i$ is not allowed. Constraint (7) imposes that link $i \to j$ cannot be used to provide UAM services if the distance L_{ij} between skyport i and skyport j exceeds the flying range limit L^*. Constraint (8)-(10) is used to model the BLM. As defined, path $m \to i \to j \to n$ is considered by passengers in OD-pair (m, n) if and only if its observable utility is the largest among all activated multimodal paths between m and n. Here, the integral restriction on y_{mijn} is relaxed to $y_{mijn} \geq 0$ because y_{mijn} is either 0 or 1 in the optimal solution. Constraint (11) is the integrality restriction on x_i.

3 Generalized Benders decomposition

We solve [SNDP] by the generalized Benders decomposition. By projecting [SNDP] onto the (x, r)-space, we obtain a master problem with a significantly smaller decision space

$$\textbf{[MP]} \quad \max \sum_{(m,n)\in K} \Phi_{mn}(r) \quad \text{s.t.. } (4)-(6), (11) \tag{12}$$

where the function $\Phi_{mn}(r)$ is defined by the following subproblem, namely, $\forall (m, n) \in K$,

$$\Phi_{mn}(r) = \max_{y\geq 0} d_{mn} \frac{\sum_{i\in I}\sum_{j\in J}\pi_{mijn}y_{mijn}}{\sum_{i\in I}\sum_{j\in J}\pi_{mijn}y_{mijn}+1} \tag{13}$$

$$\textbf{[SP]} \quad \text{st.} \sum_{i\in I}\sum_{j\in J} y_{mijn} = 1 \tag{14}$$

$$y_{mijn} \leq r_{ij} \tag{15}$$

where $\pi_{mijn} = e^{\tilde{u}(\tilde{t}_{mijn}, t_{ij})}/e^{u(t_{mn})}, \forall i, j \in I, (m, n) \in K$. Note that $\Phi_{mn}(r)$ is a concave function over r. For any fixed $\bar{r}$, we have the (globally) valid inequality for [MP]

$$\beta_{mn} \leq \Phi_{mn}(r) \leq \Phi_{mn}(\bar{r}) + \sum_{i\in I}\sum_{j\in J} \bar{S}_{mijn}(r_{ij} - \bar{r}_{ij}), \forall (m, n) \in K \tag{16}$$

where $\bar{S}_{mijn}$ is the subgradient of $\Phi_{mn}(r)$ at $\bar{r}_{ij}$. (16) is referred to as the *generalized Benders cut* (GBC). Using it, [MP] can be modeled by the following MILP (relaxed master problem):

$$\max \sum_{(m,n)\in K} \beta_{mn} \tag{17}$$

$$\textbf{[rMP]} \quad \text{st. } \beta_{mn} \leq \Phi_{mn}(\bar{r}) + \sum_{i\in I}\sum_{j\in J} \bar{S}_{mijn}(r_{ij} - \bar{r}_{ij}), \forall (m, n) \in K, \bar{r} \in \Xi \tag{18}$$

$$(4)-(6), (11)$$

where Ξ is the set of integer points r, which define GBCs that are generated by evaluating $\Phi_{mn}(r)$ at some r values and are then added to [rMP] as needed to cut off non-optimal solutions.

The main challenge to solve [SNDP] using [rMP] is how to generate GBCs, or more specifically, how to obtain the subgradient $\bar{S}_{mijn}$ at $\bar{r}$. Here we present an analytical approach. Given the primal solution $(\bar{r}, \bar{y})$, we can obtain the dual values by the following lemma.

Lemma 1. *Suppose $\bar{r}$ is an integer solution from [rMP]. For OD-pair (m, n), let $(\hat{i}, \hat{j})$ be such that $y_{m\hat{i}\hat{j}n} = 1$. The subgradient $\bar{S}_{mijn}$ can be computed as*

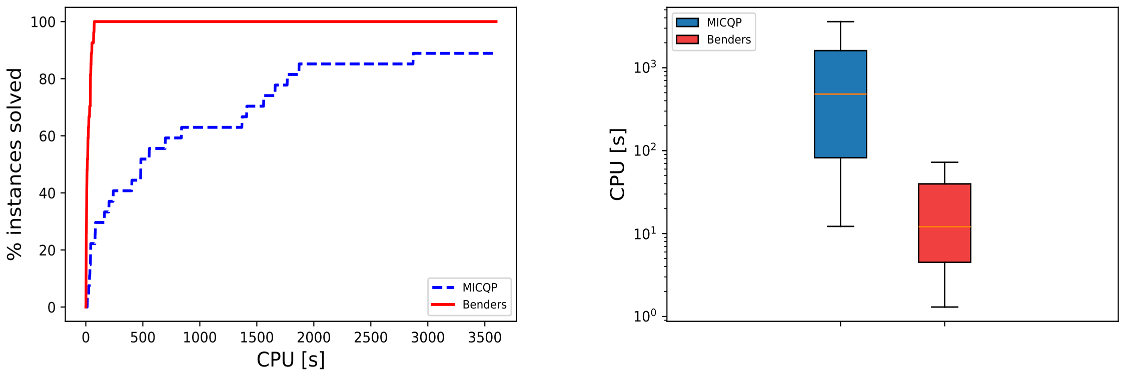
$$\bar{S}_{mijn} = (1 - \bar{r}_{ij})d_{mn} \left[\frac{\pi_{mijn} - \pi_{m\hat{i}\hat{j}n}}{(\pi_{m\hat{i}\hat{j}n} + 1)^2}\right]_+ \quad \forall i, j \in I \tag{19}$$

where $[x]_+ = \max\{x, 0\}$.

With the above result, [rMP] is ready to be solve branch-and-cut approaches.

4 Numerical experiment

In this section, we conduct numerical experiments to test the proposed Benders approach. As a benchmark, we consider solving [SNDP] by a mixed-integer conic quadratic program (MICQP) approach. We use a randomly generated dataset. Here, $e^{\tilde{u}(\tilde{t}_{mijn},t_{ij})} = e^{-a(L_{mi}+L_{ij}/\psi+L_{jn})-bL_{ij}/\psi}, \forall i,j \in I, (m,n) \in K$ and $e^{u(t_{mn})} = \xi_{mn} \cdot e^{-aL_{mn}}, \forall (m,n) \in K$, where L_{mi} is the distance between origin m and skyport i; L_{ij} is the distance of link $i \to j$; L_{jn} is the distance from skyport j to destination n; and L_{mn} is the direct distance between origin m and destination n. The distance is measured in 100 meters. For simplicity, ψ and ξ_{mn} are fixed at 3 and 2. By varying parameters (a, b, B) and considering three sizes of networks, i.e., $(|K|, |I|) \in \{(208, 20), (362, 25), (538, 30)\}$, we create 54 problem instances. We set a time limit of 3600 seconds for solving these instances. Figure 2 summarizes the computational results. Clearly, our proposed Benders approach outperforms MICQP by a large margin: the computational time by MICQP is generally more than one order of magnitude higher. Benders-based approaches can efficiently solve all instances with a small amount of computational time.



(a) CPU versus Percentage of instances solved

(b) Boxplot of CPU in log-scale

Figure 2 – *Computational results of 54 instances.*

5 Conclusion

We studied the strategic planning of the UAM system where the OD demand and passengers' route choices were characterized by a discrete choice model. We aimed to maximize the ridership. A branch-and-cut algorithm was developed to solve the problem in a reasonable time. In the future, we will present case studies to demonstrate the usefulness of our proposed framework.

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