# Dynamic management of shared autonomous electric vehicles and charging bays considering battery swapping queue delays

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## 1 INTRODUCTION

The electrification and automation of carsharing services are poised to revolutionize the industry. Shared autonomous electric vehicles (SAEVs) represent an emerging mobility mode with the potential to significantly reduce labour costs and greenhouse gas emissions. However, most existing studies have focused on plug-in charging as the primary refuelling method (Dong *et al.*, 2022), while battery swapping, an efficient alternative particularly suited for commercial EVs, has received limited consideration. Battery swapping enables rapid energy replenishment within minutes and enhances fleet management efficiency, making it a compelling option for SAEVs.

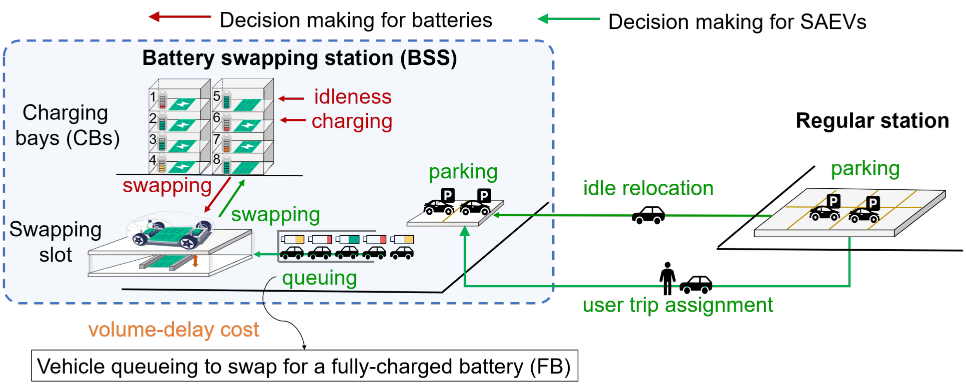


Figure 1 – *Illustration of the operational decisions for the SAEV system at given time of day*

Daily management of SAEV systems with battery swapping requires balancing vehicle supply with user demand across stations and optimizing battery charging and swapping schedules at each battery swapping station (BSS) (Shen *et al.*, 2019). Limited availability of fully charged batteries (FBs) and swapping slots leads to inevitable queuing at BSSs (Ding *et al.*, 2022), which, if not considered in dynamic operations, can cause excessive vehicle dwelling times, inefficient battery swaps, and worsened supply-demand imbalances, thereby increasing operational costs and reducing demand satisfaction rates. Figure 1 illustrates the operational process of SAEV systems, including SAEVs, regular stations, and BSSs, highlighting decision-making processes and the delay costs due to vehicle queues. Despite advancements in modelling and relocation strategies for BSSs (Cui *et al.*, 2023), the joint optimization of battery management and vehicle operations, particularly considering vehicle queue delays, remains underexplored. To address these challenges, we propose a dynamic SEAV and battery in charging bays (CBs) management

(VBM) problem using dual time-electricity-expanded networks to optimize the battery charging schedules in CBs at each BSS and vehicle relocation, considering battery swapping queue delays which adhere to the first-in, first-out (FIFO) principle due to space constraints at BSSs.

# 2 METHODOLOGY

This paper aims to optimize the operational decisions of a SAEV system with battery swapping. To maximize the profit, the system operator makes optimal user trip assignment, idle vehicle relocation, battery charging, and vehicle and battery swapping decisions for finite time periods $\mathcal{T} = \{1, \cdots, |\mathcal{T}|\}$. During the battery swapping process, the operator selects vehicles with low State of Charge (SoC) to swap with FBs available at the BSSs. The SoC of batteries in SAEVs and CBs is discretized into multiple levels $\mathcal{E} = \{1, ..., |\mathcal{E}|\}$. The service region is divided into stations $\mathcal{I} = \{i, j, \cdots\}$, categorized into regular stations, $\mathcal{I}_1 \subset \mathcal{I}$, and BSSs, $\mathcal{I}_2 \subset \mathcal{I}$. Each BSS consists of a set of CBs, $\mathcal{B}_i = \{1, \cdots, |\mathcal{B}_i|\}$ for $i \in \mathcal{I}_2$, and is equipped with one swapping slot and a finite queue length. We assume that the maximum allowable delay time at a BSS is $W$ determined by the queue length, and each battery swapping takes $s$ minutes.

To capture the dynamic operational characteristics of the SAEV system, we propose a network model based on two time-electricity-expanded networks, which are coupled through vehicle and battery swapping. One network models the larger-scale SAEV movements between stations, as well as parking and swapping at stations. The other network models battery management within CBs at each BSS, aiming to determine an optimal *battery schedule path* for each CB, which represents a sequence of dynamic battery management in a CB such as charging, idleness, and swapping. To account for vehicle queuing, we introduce link capacity constraints related to queuing flows, ensuring that the FIFO principle holds. The VBM problem is then formulated as a mixed-integer linear programming (MIP) model, integrating both vehicular flows and battery schedule paths of CBs within the same framework. To address this NP-hard problem, we customize an exact branch-and-price (B&P) algorithm, demonstrated to outperform greatly commercial solvers such as Gurobi in the computational study.

## 2.1 Network representations

We define two time-electricity-expanded networks to model the operational dynamics.

**Battery-related network**. As shown in **Figure** 2(a), for CBs, let $G^{\mathrm{c}}(\mathcal{N}^{\mathrm{c}}, \mathcal{A}^{\mathrm{c}})$ denote a time-electricity-expanded network. Each node $b_i^{t,e} \in \mathcal{N}^{\mathrm{c}}$ represents a battery with SOC level $e \in \mathcal{E}$ stored in CB $b \in \mathcal{B}_i$ at BSS $i \in \mathcal{I}_2$ during time period $t \in \mathcal{T}$. Each link $a = (b_i^{t,e}, b_i^{t+1,e'}) \in \mathcal{A}^{\mathrm{c}}$ is associated with three attributes, BSS $i$, CB $b$, start-end SoC difference $e_a = e - e'$. The set of all battery-related links is $\mathcal{A}^{\mathrm{c}} = \mathcal{A}^{\mathrm{c}}_{\text{charge}} \cup \mathcal{A}^{\mathrm{c}}_{\text{swap}} \cup \mathcal{A}^{\mathrm{c}}_{\text{idle}}$, where $e_a < 0$ for $\mathcal{A}^{\mathrm{c}}_{\text{charge}}$, $e_a = |\mathcal{E}| - e > 0$ for $\mathcal{A}^{\mathrm{c}}_{\text{swap}}$, and $e_a = 0$ for $\mathcal{A}^{\mathrm{c}}_{\text{idle}}$. The binary decision variable $f_p^b \in \{0, 1\}$ indicates whether battery schedule path $p \in \mathcal{P}$ is selected for CB $b \in \mathcal{B}_i$. The indicator $\delta^b_{a,p}$ equals 1 if link $a \in \mathcal{A}^{\mathrm{c}}$ is part of path $p$ for CB $b$, and 0 otherwise.

**Vehicle-related network**. As illustrated in **Figure** 2(b), for SAEVs, let $G^{\mathrm{v}}(\mathcal{N}^{\mathrm{v}}, \mathcal{A}^{\mathrm{v}})$ denote a time-electricity-expanded network. The nodes $\mathcal{N}^{\mathrm{v}} = \mathcal{N}^{\mathrm{v}}_1 \cup \mathcal{N}^{\mathrm{v}}_2$ consist of move-park nodes $\mathcal{N}^{\mathrm{v}}_1 = \{n_i^{t,e}, \forall i \in \mathcal{I}, t \in \mathcal{T}, e \in \mathcal{E}\}$, which represent vehicle locations with time-electricity tags for movement and parking, and swap-queue nodes $\mathcal{N}^{\mathrm{v}}_2 = \{n_{iw}^{t,e}, \forall i \in \mathcal{I}_2, t \in \mathcal{T}, e \in \mathcal{E}, w \in T \backslash \{\mathcal{T} - W + 1, \mathcal{T}\}$, which involve the vehicle swapping and queueing processes. Correspondingly, the links in $\mathcal{A}^{\mathrm{v}} = \mathcal{A}^{\mathrm{v}}_1 \cup \mathcal{A}^{\mathrm{v}}_2$ consist of regular links $\mathcal{A}^{\mathrm{v}}_1 = \mathcal{A}^{\mathrm{v}}_{\mathsf{trip}} \cup \mathcal{A}^{\mathrm{v}}_{\mathsf{relo}} \cup \mathcal{A}^{\mathrm{v}}_{\mathsf{park}} \cup \mathcal{A}^{\mathrm{v}}_{\mathsf{dummy}}$, representing vehicle trips, relocation, parking, and dummy actions. Each link $a = (n_i^{t,e}, n_j^{t',e'}) \in \mathcal{A}^{\mathrm{v}}_1$ corresponds to either vehicle movement between stations ($i \neq j$, $t' = t + t_a$, $e_a > 0$), or parking at the station ($i = j$, $t' = t + 1$, $e_a = 0$). Additionally, swap-queue-related links $\mathcal{A}^{\mathrm{v}}_2 = \mathcal{A}^{\mathrm{v}}_{\mathsf{swap}} \cup \mathcal{A}^{\mathrm{v}}_{\mathsf{queue}} \cup \mathcal{A}^{\mathrm{v}}_{\mathsf{prior}}$ represent vehicle swapping, queueing, and priority-dummy actions. Each link $a = (n_{iw}^{t,e}, n_{iw}^{t',e}) \in \mathcal{A}^{\mathrm{v}}_2$ captures swapping or queueing processes. The vehicle-related decision variables are the flow rates $x_a$ on the links, where $x_a^{\mathsf{T}}$, $x_a^{\mathsf{R}}$, $x_a^{\mathsf{P}}$, $x_a^{\mathsf{S}}$, $x_a^{\mathsf{Q}}$, and $x_a^{\mathsf{PD}}$ denote the flow rates for user trips, relocation, parking, swapping, queueing, and priority-dummy actions, respectively, on their corresponding link sets $\mathcal{A}^{\text{v}}_{\text{trip}}$, $\mathcal{A}^{\text{v}}_{\text{relo}}$, $\mathcal{A}^{\text{v}}_{\text{park}}$, $\mathcal{A}^{\text{v}}_{\text{swap}}$, $\mathcal{A}^{\text{v}}_{\text{queue}}$, and $\mathcal{A}^{\text{v}}_{\text{prior}}$.

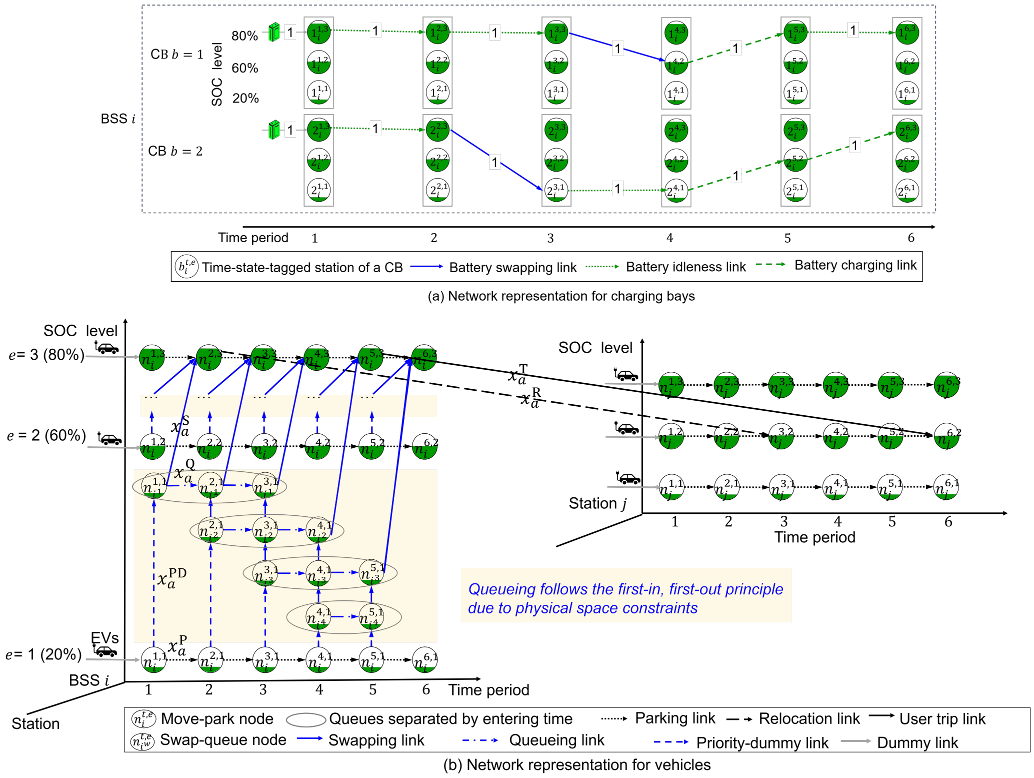


Figure 2 – *Illustration of two time-electricity-expanded networks for charging bays and vehicles*

## 2.2 Mixed-integer linear programming model formulation

Building on the above definition and notations, we cast the dynamic management problem of SAEVs and CBs as an MIP model:

$$\min_{\mathbf{x}\geq\mathbf{0},\mathbf{f}\in\{0,1\}} \; -\sum_{a\in\mathcal{A}^{\text{v}}_{\text{trip}}} p_a x^{\text{T}}_a + \sum_{a\in\mathcal{A}^{\text{v}}_{\text{relo}}} c^{\text{r}}_a x^{\text{R}}_a + \sum_{a\in\mathcal{A}^{\text{v}}_{\text{park}}} c^{\text{P}} x^{\text{P}}_a + \sum_{a\in\mathcal{A}^{\text{v}}_{\text{queue}}} c^{\text{q}} x^{\text{Q}}_a + \sum_{i\in\mathcal{I}_2}\sum_{b\in\mathcal{B}_i}\sum_{p\in\mathcal{P}} c_p f^b_p \tag{1}$$

$$\text{s.t. } \mathbf{A}\mathbf{x} = (V_{i,e}, \mathbf{0})^{\text{T}} \;\; \forall i\in\mathcal{I}, t\in\mathcal{T}, e\in\mathcal{E}, \tag{2}$$

$$\sum_{p\in\mathcal{P}} f^b_p = 1 \;\; \forall i\in\mathcal{I}_2,\, b\in\mathcal{B}_i, \tag{3}$$

$$x^{\text{S}}_a = \sum_{b\in\mathcal{B}_i}\sum_{p\in\mathcal{P}} (f^b_p \delta^b_{a',p}) \;\; \forall a\in\mathcal{A}^{\text{v}}_{\text{swap}_1}, a'\in\mathcal{A}^{\text{c}}_{\text{swap}}, \tag{4}$$

$$x^{\text{T}}_a \leq D_{i,t,e} \;\; \forall a\in\mathcal{A}^{\text{v}}_{\text{trip}}, \;\; \sum_{e\in\mathcal{E}} x^{\text{P}}_a \leq C_i \;\; \forall a\in\mathcal{A}^{\text{v}}_{\text{park}}, \text{ and } \sum_{e\in\mathcal{E}} x^{\text{S}}_a \leq \lfloor\triangle\mathcal{T}/s\rfloor \;\; \forall a\in\mathcal{A}^{\text{v}}_{\text{swap}_1}, \tag{5}$$

$$\sum_{e\in\mathcal{E}} x^{\text{S}}_a \leq \lfloor\triangle\mathcal{T}/s\rfloor - \sum_{a'\in S_{a'}}\sum_{e\in\mathcal{E}} x^{\text{S}}_{a'} \;\; \forall a\in\mathcal{A}^{\text{v}}_{\text{swap}_2}, \tag{6}$$

where objective function (1) aims to minimize net loss (i.e., maximize profits), obtained by subtracting total revenue from vehicle relocation, parking, queuing, and battery charging costs. Eq. (2) enforces vehicular flow conservation, Eq. (3) optimizes battery schedules for each CB, and Eq. (4) couples vehicles and batteries for swapping. To model the queueing process within the network $G^{\text{v}}$, $\mathcal{A}^{\text{v}}_{\text{swap}}$ is divided into actual $\mathcal{A}^{\text{v}}_{\text{swap1}} = \{a = (n^{t,e}_{iw}, n^{t+1,|\mathcal{E}|}_i)\}$, and possible swapping links $\mathcal{A}^{\text{v}}_{\text{swap2}} = \{a = (n^{t,e}_{iw}, n^{t,e}_{iw-1})\}$. For a possible swapping link $a = (n^{t,e}_{iw}, n^{t,e}_{iw-1}), a\in\mathcal{A}^{\text{v}}_{\text{swap2}}$, define $S_a$ as the set of queuing links with $a' = (n_{iw'}^{t'-1,e}, n_{iw'}^{t',e}), t' = t, 1 \le w' \le w-1, a' \in \mathcal{A}_{\text{queue}}^{\text{v}}$. Eqs. (5) impose side constraints on flow types, while Eq. (6) ensures FIFO compliance by limiting swapping link capacity based on queuing flows.

### 2.3 Branch-and-price algorithm

The proposed large-scale MIP model, replete with numerous path-based variables, presents a complex challenge due to its coefficient matrix and vehicle flow constraints not being totally unimodular, which hinders straightforward solutions using solvers. Inspired by its intrinsic block-diagonal structure, we customize an exact solution method, B&P algorithm, significantly improving the computational efficiency. The procedure of the B&P algorithm is summarized as follows,

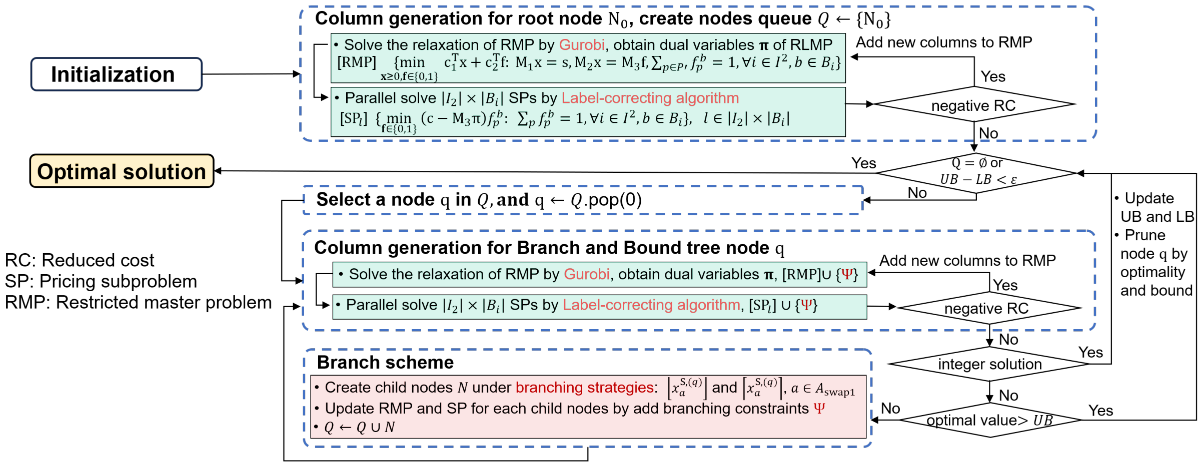


Figure 3 – *The flowchart of the customized branch-and-price algorithm*

## 3 CONCLUSION

Extensive numerical experiments conducted on large-scale instances generated from realistic data in Shanghai highlight the superior computational performance of the proposed B&P algorithm relative to the benchmarking methods. Sensitivity analyses are conducted on parameters such as the number of BSSs and CBs, battery swapping duration and time-of-use (ToU) electricity prices. Numerical results show that ToU prices have a significant impact on the battery of CB charging schedules and vehicle operations, with an increase in FB inventory during low-price periods. Additionally, waiting queues at BSSs tend to have more vehicles with lower SoC levels, as queuing becomes a more cost-effective option under such conditions. Overall, the proposed model and customized algorithm offer a management tool for dynamic vehicle operations and battery management of the emerging SAEV systems in practice.

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