Integrated Urban Transportation Network Design for Alleviating Transit Deserts

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1 INTRODUCTION AND CONTRIBUTIONS

Urbanization amplified the prevalence and severity of gaps in public transit access. The term "transit desert" describes an area of the city where the transit demand exceeds supply. In some cities, more than 1 in 8 residents live in transit deserts. They are often forced to choose between walking long distances to access nearest transit stop or using alternative means of transportation such as cars, taxis, and ride-hailing services. These alternatives produce more greenhouse gas emissions, exacerbate road congestion, and are not affordable for some residents, leading to financial distress, job losses, and reduced socioeconomic mobility. Many transit desert residents lack access to fresh food, clean air, and medical care. Transit deserts often coincide with residences of lower-income and minority populations, exacerbating income and racial inequalities.

Transit agencies lack incentives or are not well equipped to extend their networks to transit deserts. Many transit agencies are struggling with financial difficulties and planning service cuts. Thus, improving transit access without a surging budget is a challenge. Ride-hailing services can improve mobility for people without access to traditional fixed-route transit (FRT). However, some studies have shown that these transportation network companies (TNCs) have had an insignificant effect on car ownership, but instead have led to a significant decline in transit ridership and worsening congestion. An alternative is to use on-demand ride-sharing (ODRS) services as feeders to FRT. Indeed, some cities have focused on integrating ODRS with FRT to help address the first/last-mile limitations of FRT. Integration could potentially improve accessibility for transit desert residents without exacerbating traffic congestion in city centers. However, governments and transit agencies currently have limited analytical tools to optimize the impacts of such projects. Importantly, most agencies have not fine-tuned their current FRT systems to maximize the benefits of ODRS integration. This paper aims to address these gaps, providing a rigorous analytical framework based on prescriptive models and algorithms to help alleviate transit deserts in a holistic way. Our framework can be seen as a tool for optimizing the use of investments in transportation infrastructure, such as the recent $1 trillion infrastructure bill in the US of which $39 billion were devoted to expanding the current transit systems.

Our first contribution is a new optimization model to determine FRT lines, frequencies, and ODRS fleet size, while explicitly capturing passengers' mode choice decisions, in order to minimize total system-wide costs. It embeds a general attraction discrete choice model that captures passenger mode choice as a function of trip expenditure, in-vehicle time, wait time, and walking time. Passengers choose between FRT, ODRS, multi-modal trips (combining FRT and ODRS), and an outside option (OO). The overall problem is a non-convex Mixed-Integer Non-Linear Optimization (MINLO) model. Existing studies either do not optimize the FRT network, or only revise it incrementally, or do not scale to real-world case studies. In contrast, ours is a general framework for jointly designing real-world FRT and ODRS networks.

Our second contribution is an original two-phase heuristic to solve the non-convex MINLO problem. The first phase selects the transit lines from a large set of candidate lines by proposing a new domain-inspired relax-and-round heuristic to solve the integrated model. The second phase decides the frequency of all selected lines and the ODRS fleet size by iteratively solving a mixed-integer second-order conic optimization model, through repeated linearizations of the non-convex constraints using first-order Taylor series expansion. The second phase is enhanced by a delayed constraints generation method to reduce the number of constraints. This two-phase heuristic consistently outperforms all algorithmic benchmarks in terms of total system-wide costs.

Finally, we generate practical insights through computational experiments for transit desert regions in the Greater Boston area. Our solutions provide a system-wide cost reduction of 2.8%-7.2%, translating into annual savings of more than $5 million. Our approach especially reduces the commuting disutility for lower-income households and households without car ownership, by 16% and 25%, respectively, resulting in a more accessible and equitable urban public transportation system. We show that the system-wide cost can be reduced further, by up to 7%, if the pricing of the ODRS service can be adjusted. Compared to distance-based pricing, flat pricing structure generates larger profits while maintaining a similar level of service.

2 METHODOLOGY

The objective function of our non-convex MINLO model is to minimize the sum of costs to passengers, operators, and the environment. The cost to passengers is the inconvenience cost of walking, in-vehicle time, and wait time, for FRT, ODRS, and multi-modal alternatives, plus the cost of using the OO. The cost to operators is the sum of the ownership, maintenance and operating costs of both the FRT and ODRS fleets. The environmental costs are due to emissions from the OO, FRT, and ODRS operations. The main constraints in the model are as follows.

1. The FRT constraints ensure that the number of transit lines is within the allowable range; upper and lower limits on the line frequencies are enforced; the size of the FRT fleet is enough to operate the line frequencies; and the overall vehicle purchase budget is respected.
2. Passenger path selection constraints ensure that a path can be used only if all FRT lines in it are selected; the shortest distance path among all available paths in that origin-destination (OD) pair is used by passengers; and at most one FRT path is used in each OD pair (enforced separately for transit-only and multi-modal paths, in each OD pair).
3. Wait times for each FRT leg of a path are set equal to the inverse of twice the frequency, and the total path wait time is the sum of wait times for each leg. These non-linear constraints (due to the reciprocal function) can be reformulated using second-order conic functions.
4. Passenger utility is the weighted sum of in-vehicle, walking, wait time, and travel expenses.
5. The mode share constraints calculate passenger flows using the sales-based linear programming reformulation of a general attraction passenger choice model (Gallego *et al.*, 2015).
6. For ODRS paths and ODRS components of multi-modal paths, a multiplier called the "detour ratio" captures the ODRS wait and detour times. ODRS travel time constraints calculate this detour ratio as a convex function of ODRS fleet size and demand density (Daganzo & Ouyang, 2019), and it can be approximated by a piecewise linear function.
7. Capacity constraints ensure that number of passengers cannot exceed FRT vehicle capacity.

2.1 Solution Approach

This model presents three computational challenges. First, real-world problem instances are very large. Our experiments had over 4,000 OD pairs, 14,000 paths, and 600,000 decision variables, of which over 65,000 were binary. Second, the objective function and some constraints have bilinear terms that involve products of continuous decision variables. Finally, some constraints also involve exponential terms. MINLO generalizes both mixed-integer optimization (MIO) problems and non-linear optimization (NLO) problems, making it particularly difficult to solve in both classes. We strive to separate the complexity of MIO from the NLO and conquer the two challenges individually by repeatedly searching for better integer solutions and solving the NLO subproblem for each case (Nannicini & Belotti, 2011), using an original two-phase approach.

The first phase is a construction heuristic based on an iterative relax-and-round strategy. This phase differs from previous studies in two aspects. First, we use a relaxation that is particularly suitable for the non-convex MINLO model, and outperforms the more standard continuous relaxation. Second, we propose a new domain-inspired rounding approach, which outperforms rounding to the nearest integer.

The previous studies solved the continuous relaxation of the original model and then fixed a subset of decision variables based on certain criteria. However, for our non-convex MINLO model, the runtime for continuous relaxation is extremely long. Due to the existence of exponential and bilinear constraints, only a few integer optimization solvers (such as Gurobi 9.0 or later) are able to handle such models. They typically introduce additional integer variables when applying piecewise linear approximation to these non-convex constraints. The resulting models are extremely hard to solve — for our instances, it took over 10 days to obtain a solution within the 5% optimality gap. Second, solvers can provide much tighter lower bounds when solving the original MINLO model than its continuous relaxation, partly due to advanced presolving and cutting-plane techniques. When solving the original model, we observe that many binary variables that should be assigned to 0 are already very close to 0 at the root node. Therefore, rather than applying the rounding method to the continuous relaxation of the original MINLO model, we feed the original MINLO model to the solver and apply the rounding method to the line selection variables obtained at the root node of the branch-and-bound tree.

A common rounding approach is to round to 1 the binary variables with the highest values at the relaxed problem's optimal solution. This method is likely to work well if the transit lines with large fractional values of the line selection variables in the relaxed problem's optimal have a high likelihood of being selected in the original MINLO model's optimal solutions. Notably, this approach does not account for the importance of the selected lines. Transit lines differ in terms of the impact of their presence in a solution. The importance of a line in a solution can be partly explained by the number of passengers carried by that line — the more passengers carried, the greater the importance of its presence. Inspired by this domain-specific insight, we develop a new rounding approach, which rounds to 1 the selection variables for the lines that carry the highest number of passengers in the relaxed problem's optimal solution. Our computational results show that this domain-inspired rounding consistently outperforms conventional rounding.

The second phase uses a first-order approximation (FOA) to iteratively improve the incumbent solution. Unlike previous transit optimization studies, our MINLO problem involves a large number of binary decision variables that make the optimization model difficult to solve even when using the FOA. Therefore, rather than applying the FOA method directly, we first apply the relax-and-round heuristic to determine the transit lines, thus eliminating a large number of integer variables. In the second phase, we apply the FOA method to optimize the mixed-integer second-order conic optimization model, while iteratively updating the reference points until convergence. We iteratively add only the violated constraints describing the convex piecewise-linear relationship of detour ratio, fleet size, and demand density to keep the model size to a minimum.

3 RESULTS

The Massachusetts Department of Transportation has identified nine regions in the Greater Boston area with low transit accessibility and a high proportion of households with lower income and without cars. We use three of these regions, which are served by buses, for our case studies.

Table 1 compares the computational performance of our proposed approach (RR-DIR-FOA) with various conventional benchmarks. Five of the benchmarks (Direct, CR-NIR, CR-NIR-FOA, RR-NIR, RR-DIR) cannot provide a feasible solution within the runtime limit for at least one of the two test cases. Although the other two benchmarks (FOA and RR-NIR-FOA) are able to provide a feasible solution, their solutions are 35.1% and 4.3% worse than the solutions of our method for the large test case. This shows the computational superiority of our approach.

3.1 Insights from Practical Case Study

At an aggregate level, compared to the status quo, our approach reduces total systemwide costs by 2.8%, 3.8% and 7.2%, in the three regions. The passengers' average disutilities also decrease

Table 1 – *Comparisons with Algorithmic Benchmarks. Gaps are w.r.t. best available solutions. Direct: Directly using Gurobi. FOA: First Order Approximation. CR: Continuous Relaxation. DIR: Domain-Inspired Rounding. RR: Root node Rounding. NIR: Nearest Integer Rounding.*

	Medium			Large		
Algorithm	Obj Val	Gap (%)	T (min)	Obj Val	Gap (%)	T (min)
Direct	NA	NA	360	NA	NA	360
FOA	49,051	5.23	360	69,653	35.08	180
CR-NIR	NA	NA	360	NA	NA	360
CR-NIR-FOA	NA	NA	360	NA	NA	360
RR-NIR	49,535	6.27	360	NA	NA	360
RR-NIR-FOA	46,899	0.62	16	53,798	4.33	96
RR-DIR	49,812	6.87	360	NA	NA	360
RR-DIR-FOA	46,612	**0.00**	16	51,565	**0.00**	92

by 5.7%, 6.6% and 10.1%, respectively. Our approach also improves the outcomes from an equity standpoint. The status quo in these transit deserts highlights the inequalities that exist between and within regions. Salem-Lynn region has the highest proportion of lower-income households (19%), while Waltham has the lowest (14%). Salem-Lynn region also has the highest number of households without cars (17%). Our solutions improved transit ridership in all three regions.

In the Woburn-Melrose region, transit ridership increases by 3 percentage points (pp), from 8.5% to 11.7%. For households without cars and with lower income, it increases more steeply, by 11.8 pp and 7.6 pp. The overall market share for multi-modal services also increases from 1.2% to 2.0% showing that a carefully designed transit network can help in the integration of FRT and ODRS services. The disutility per person decreases from \$18.3 to \$17.2. We find that commuters without cars have the largest decrease in travel disutility from \$40.5 to \$30.5, from switching to lower-cost FRT alternative instead of having to use the expensive outside options. Lower-income households also receive a significant decrease in disutility from \$15.8 to \$13.3, while disutility for higher-income households only decreases slightly by \$0.78. Thus, our solutions are especially beneficial to underprivileged communities. The results are qualitatively similar in the other two regions as well. The total number of cars on the roads during the morning rush hours was reduced by 103 (2.6%), 95 (3.3%), and 159 (10.1%), in Woburn, Salem-Lynn, and Waltham, respectively. Therefore, road congestion is also expected to improve under our proposed solution. The environmental costs are also lower in our solution compared to the status quo. Waltham has the highest percentage reduction of 4% while Woburn has the largest absolute decrease. In general, we find that an improved transportation network helps reduce greenhouse gas emissions.

4 DISCUSSION

We contribute a non-convex MINLO model for jointly optimizing FRT lines, frequencies, and ODRS fleet size, while accounting for passenger choice. Our original two-phase solution method combines a domain-inspired relax-and-round approach, iterative first-order approximations, and delayed constraints generation. Our approach consistently outperforms algorithmic benchmarks and provides substantial improvements through lower total costs, higher passenger benefits, lower emissions, and enhanced equity in practical case studies, thus pareto dominating the status quo.

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