# The Scheduled Service Network Design Problem with Bin Packing and Heterogenous Fleets

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*Extended abstract submitted for presentation at the 12th Triennial Symposium on Transportation Analysis conference (TRISTAN XII)*
*June 22-27, 2025, Okinawa, Japan*

February 12, 2025

Keywords: freight transportation, consolidation, integer programming, branch and price

## 1 Introduction

In many supply chain contexts shipments are transported that are small relative to the capacity of the transporting vehicle. Given that the majority of transportation costs are incurred on a per-vehicle basis these costs exhibit economies of scale. Namely, the cost per shipment transported decreases as the number of shipments in a vehicle increases. Thus, in contexts where revenues are earned on a per shipment basis, high vehicle utilization is critical to profitability. Examples of such contexts include an eCommerce retailer fulfilling customer orders by transporting goods through a distribution network and a third party transportation carrier that quotes prices on a per-unit-of-vehicle capacity basis.

The primary strategy for achieving high vehicle utilization in these contexts is to route shipments on paths through a network of terminals. Doing so facilitates synchronizing the paths of shipments from different customers, with potentially different initial origins and final destinations, to include the same terminal to terminal movements. Further synchronizing the schedules of these shipment paths enables the consolidation of such shipments in the same vehicle dispatch. Thus, one lever for reducing per-shipment transportation costs is to design and schedule the paths of shipments to be synchronized in both space and time. However, when allocating vehicle capacity to transport shipments there may be choices regarding the type of vehicle used, with different vehicle having different capacities and costs. Thus, another lever for reducing per-shipment transportation costs is to rightsize the capacity allocated to a transportation move.

This research focuses on a Mixed Integer Programming (MIP) methodology for planning the operations of such carriers that leverages both cost-reduction levers. The methodology is based on solving a variant of the Scheduled Service Network Design Problem (SSNDP) that we refer to as the Scheduled Service Network Design Problem with Bin Packing and Heterogenous Fleet (SSNDP-BPHF). The SSNDP has received extensive study in the transportation and logistics literature (Crainic & Hewitt, 2021). Further, some research has considered variants in which the fleet of vehicles available to transport shipments is heterogenous (Wang *et al.*, 2019). However, nearly all research focused on the SSNDP has modeled capacity in aggregate. Namely, it has ignored the operational reality that individual shipments must be allocated to individual vehicles and the shipments allocated to a vehicle must fit within that vehicle. Hewitt & Lehuédé (2023) presents a new MIP formulation of the SSNDP that facilitates modeling such bin packing considerations and illustrates that ignoring them can understimate transportation costs by up to 7.5%, with the underestimate increasing in the number of shipments to be transported. However, that formulation is enumerative in nature and has a potentially exponential number of variables. As

Hewitt & Lehuédé (2023) does not present an algorithm for addressing that issue, only instances with small numbers of commodities are considered.

This research includes multiple contributions related to the literature on freight transportation in general and the SSNDP in particular. Namely, it includes a MIP formulation of the SSNDP that is both provably stronger than the one presented in Hewitt & Lehuédé (2023) and easily adaptable to the SSNDP-BPHF. It also includes a Branch-and-price algorithm for solving larger instances of that formulation than those considered in Hewitt & Lehuédé (2023).

# 2 Methodology

We begin with a mathematical definition of the operational context considered by the SSNDP-BPHF. The terminal network is modeled as a directed network $\mathcal{D} = (\mathcal{N}, \mathcal{A})$ in which the set $\mathcal{N}$ models consolidation terminals and the set $\mathcal{A}$ models physical transportation moves between terminals. Regarding transportation, there is a set of vehicle types, $\mathcal{G}$, such that the cost of dispatching a vehicle of type $g \in \mathcal{G}$ on arc $(i, j) \in \mathcal{A}$ is $f^g_{ij}$. Relatedly, a vehicle of type $g$ has capacity $u^g_{ij}$ on that arc. We do not model that vehicles of different types travel at different speeds. Thus, the time needed for a vehicle to travel on arc $(i, j)$ is given by $\tau_{ij}$.

There is a set of shipments $\mathcal{K}$ that require transportation. Associated with each shipment $k \in \mathcal{K}$ is a terminal where it is to be picked up, $o_k$, no earlier than the release date $e_k$, and is to be delivered, $d_k$, no later than the due date $l_k$. In addition, associated with shipment $k$ is its size $q_k$, expressed in the same unit as vehicle capacity. Lastly, associated with commodity $k$ is a set of potential paths, $P_k$, on which the commodity can be routed from $o_k$ to $d_k$. We let $P_k(i, j)$ denote the set of such paths that contain arc $(i, j) \in \mathcal{A}$.

Classical formulations of the SSNDP rely on a time-expanded network to capture the scheduling of commodity and vehicle dispatches on arcs. Knapsack-type linking constraints formulated on arcs in that network ensure sufficient vehicle capacity, in aggregate, is allocated. It is well known that such inequalities lead to notoriously weak linear programming realaxations. They are also not amenable to modeling bin packing without the addition of additional variables.

Hewitt & Lehuédé (2023) propose formulating the SSNDP with *consolidations*, wherein a consolidation $\omega \subseteq \mathcal{K}$ for a given arc $(i, j)$ indicates a set of commodities that dispatch at the same time. Thus, one can compute and associate with consolidation $\omega$ a coefficient $s_\omega$ that dicates the number of vehicles it requires. Further, one can then formulate the scheduling dimension of the problem with continuous variables that model when a commodity dispatches on an arc, obviating the need for a time-expanded network. Big-M constraints triggered by the values of decision variables associated with consolidations ensure the commodities in a chosen consolidation for a given arc dispatch at the same time. Such a formulation can be proven to have a stronger linear relaxation than the classical formulation of the SSNDP based on a time-expanded network and knapsack-type constraints. It also facilitates modeling bin packing considerations, as doing so only requires appropriately computing the values $s_\omega$.

We build off the concept of consolidations to propose a formulation of the SSNDP with a linear relaxation that yields an even stronger bound. This formulation is based on the concept of *consolidation profiles*, wherein a consolidation profile $\pi = \{\omega_1, \ldots, \omega_{n_\pi}\}$ for a given arc indicates the complete set of consolidations chosen for that arc. Associated with profile $\pi$ is a binary indicator $\phi^k_\pi, k \in \mathcal{K}$ of whether commodity $k$ is contained in a consolidation in profile $\pi$ as well as another binary indicator $\sigma^\pi_{kk'}, k, k' \in \mathcal{K}$ of whether commoditites $k, k'$ are contained in the same consolidation in profile $\pi$. The bin packing and heterogenous fleet aspects of the SSNDP-BPHF are captured in the coefficient $s^g_\pi$ associated with consolidation profile $\pi$ that indicates the number of vehicles of each type, $s^g_\pi, g \in \mathcal{G}$ it requires. We let $\Pi_{ij}$ denote the set of consolidation profiles for arc $(i, j) \in \mathcal{A}$.

Given these mathematical constructs, we formulate the SSNDP with Bin Packing and Heterogenous Fleet (SSNDP-BPHF) with the following decision variables. We let the binary decision variable $v^k_p, k \in \mathcal{K}, p \in P_k$ indicate whether path $p \in P_k$ is chosen for commodity $k$. We let the continuous decision variable $\gamma^k_{ij}, k \in \mathcal{K}, (i, j) \in \mathcal{A}$ indicate the time at which commodity $k$ dispatches on arc $(i,j) \in \mathcal{A}$. We let the binary decision variable $y^{\pi}_{ij}, \pi \in \Pi_{ij}, (i,j) \in \mathcal{A}$ indicate whether consolidation profile $\pi$ is chosen for arc $(i,j) \in \mathcal{A}$. Lastly, we let the integer variable $z^{g}_{ij}, (i,j) \in \mathcal{A}, g \in \mathcal{G}$ indicate the number of vehicles of type $g \in \mathcal{G}$ that are needed on arc $(i,j) \in \mathcal{A}$. We are thus able to formulate the SSNDP-BPHF as follows.

$$\text{minimize} \sum_{g \in \mathcal{G}} \sum_{(i,j) \in \mathcal{A}} f^{g}_{ij} z^{g}_{ij} \tag{1}$$

subject to

$$\sum_{p \in P_k} v^k_p = 1 \quad \forall k \in \mathcal{K}, \tag{2}$$

$$\sum_{(o_k,j) \in A} \gamma^k_{o_k j} \geq e_k \quad \forall k \in K, \tag{3}$$

$$\sum_{(i,d_k) \in A} (\gamma^k_{id_k} + \tau_{id_k}(\sum_{p \in P(i,d_k)_k} v^k_p)) \leq l_k \quad \forall k \in K, \tag{4}$$

$$\sum_{(i,j) \in A} (\gamma^k_{ij} + \tau_{ij}(\sum_{p \in P(i,j)_k} v^k_p)) \leq \sum_{(j,i) \in A} \gamma^k_{ji} \quad \forall j \in N, k \in K \tag{5}$$

$$\sum_{p \in P_k : (i,j) \in p} v^k_p = \sum_{\pi \in \Pi_{ij}} \phi^k_{\pi} y^{\pi}_{ij} \quad \forall k \in \mathcal{K}, (i,j) \in \mathcal{A}, \tag{6}$$

$$\sum_{\pi \in \Pi_{ij}} s^g_{\pi} y^{\pi}_{ij} \leq z^g_{ij} \quad \forall (i,j) \in \mathcal{A}, g \in \mathcal{G}, \tag{7}$$

$$\gamma^k_{ij} - \gamma^{k'}_{ij} \leq T(1 - \sum_{\pi \in \Pi_{ij}} \sigma^{kk'}_{\pi} y^{\pi}_{ij}) \quad \forall (i,j) \in \mathcal{A}, k, k' \in \mathcal{K} \tag{8}$$

$$\sum_{\pi \in \Pi_{ij}} y^{\pi}_{ij} \leq 1 \quad \forall (i,j) \in \mathcal{A}. \tag{9}$$

The objective function (1) minimizes total transportation costs, considering all types of vehicles. Constraints (2) ensure that each commodity is routed on a single path through the terminal network while constraints (3) - (5) ensure that the dispatch times associated with arcs on that path agree with the available and due dates of a commodity as well as travel times. Constraints (6) ensure that a consolidation profile containing a commodity is chosen for each arc in the path chosen for that commodity. Constraints (7) ensure sufficient vehicles are allocated given the consolidation profile chosen for an arc while (8) ensure that two commodities that a chosen consolidation profile indicate should dispatch at the same time do so. Finally, constraints (9) ensure at most one consolidaton profile is chosen for each arc. Omitted are constraints defining the decision variables and their domains.

Given the potentially large sizes of the sets of consolidation profiles, $\Pi_{ij}$, only small instances of the above formulation can be instantiated and solved *a priori* in reasonable runtimes. Thus, we propose solving larger instances of the formulation with a Branch-and-Price (Barnhart *et al.*, 1998) scheme. Due to space considerations, we only summarize the scheme. This scheme involves solving the linear relaxation of a Restricted Master Problem (RMP) of the same form as the formulation presented above, albeit formulated with subsets $\underline{\Pi_{ij}} \subseteq \Pi_{ij}$ of consolidation profiles for each arc. Dual variables associated with the constraints in which a $y^{\pi}_{ij}$ variable participates ((6),(7),(8),(9)) are used to formulate a pricing problem to identify variables not present in $\underline{\Pi_{ij}}$ that have negative reduced cost. That pricing problem takes the form of a heterogeneous bin packing-type problem with a quadratic term in the objective due to constraints (8). To produce optimal solutions to the integer program a branch-and-bound tree is searched with a process that solves RMPs and pricing problems at nodes within the tree. Along with branching on the binary variables $v^k_p$ and integer variables $z^g_{ij}$, the attributes $\sigma^{\pi}_{kk'}$ are branched on in a manner analogous to the Ryan & Foster branching rule (Ryan & Foster, 1981). We note further that embedded in the scheme are multiple enhancements, including primal heuristics and valid inequalities.

## 3 Results

To assess the computational efficiency of the proposed Branch-and-price scheme it was executed on the instances used in Boland *et al.* (2017). We note that to the best of our knowledge the SSNDP-BPHF has never been formulated and solved for these instances. As a benchmark, a comparable Branch-and-price scheme was developed for the formulation presented in Hewitt & Lehuédé (2023). As that formulation is not immediately amenable to heterogenous fleets, all instances consisted of a homogenous fleet (i.e. $|\mathcal{G}| = 1$). Both Branch-and-price schemes were implemented in C++ using the Branch-and-price framework of the SCIP optimization solver (Bestuzheva *et al.*, 2021). In both schemes the pricing problem was solved as a Mixed Integer Program by SCIP after linearizing the quadratic term in the objective using known techniques. As another benchmark, we solved instances of the SSNDP-BPHF formulated on a time-expanded network with SCIP. All experiments were run with a time limit of one hour and an optimality tolerance of 1%.

We report in Table 1 two summary statistics regarding the performance of the two Branch-and-price schemes, averaged over instances containing the same number of commodities. The first (# Solved) reports on the number of instances solved to within a tolerance of 1% in the allotted time limit of one hour. The second (Gap unsolved) reports the average optimality gap averaged over the instances that scheme did not solve.

Table 1 – *Results considering a homogenous fleet*

| | Time-expanded network formulation | | Consolidation formulation | | Consolidation profile formulation | |
|---|---|---|---|---|---|---|
| $\|\mathcal{K}\|$ | # Solved | Gap unsolved | # Solved | Gap unsolved | # Solved | Gap unsolved |
| 40 | 89 | 85% | 126 | N/A | 126 | N/A |
| 100 | 57 | 88% | 100 | 2.03% | 124 | 1.46% |
| 200 | 0 | 174% | 11 | 4.33% | 19 | 2.26% |
| 400 | 0 | 355% | 0 | 19.84% | 14 | 3.42% |
| Summary | 158 | 176% | 237 | 10.98% | 283 | 2.75% |

We see that applying a Branch-and-price scheme to the Consolidation profile-based formulation enables the solution of the largest number of instances. For instances not solved, the optimality gap is smaller. We also see the scheme applied to the Consolidation profile-based formulation scales better with respect to the number of commodities present in an instance than the analogous scheme applied to the Consolidation-based formulation.

## References


Barnhart, Cynthia, Johnson, Ellis L, Nemhauser, George L, Savelsbergh, Martin WP, & Vance, Pamela H. 1998. Branch-and-price: Column generation for solving huge integer programs. *Operations research*, **46**(3), 316–329.

Bestuzheva, Ksenia, Besançon, Mathieu, Chen, Wei-Kun, Chmiela, Antonia, Donkiewicz, Tim, van Doornmalen, Jasper, Eifler, Leon, Gaul, Oliver, Gamrath, Gerald, Gleixner, Ambros, *et al.* 2021. The SCIP optimization suite 8.0. *arXiv preprint arXiv:2112.08872*.

Boland, Natashia, Hewitt, Mike, Marshall, Luke, & Savelsbergh, Martin. 2017. The continuous-time service network design problem. *Operations Research*, **65**(5), 1303–1321.

Crainic, Teodor Gabriel, & Hewitt, Mike. 2021. *Service Network Design*. Springer International Publishing. Pages 347–382.

Hewitt, Mike, & Lehuédé, Fabien. 2023. New formulations for the Scheduled Service Network Design Problem. *Transportation Research Part B: Methodological*, **172**, 117–133.

Ryan, David M, & Foster, Brian A. 1981. An integer programming approach to scheduling. *Computer scheduling of public transport urban passenger vehicle and crew scheduling*, 269–280.

Wang, Zujian, Qi, Mingyao, Cheng, Chun, & Zhang, Canrong. 2019. A hybrid algorithm for large-scale service network design considering a heterogeneous fleet. *European Journal of Operational Research*, **276**(2), 483–494.