A time-slot management problem with mixed logit demand

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1 Introduction

The rise of e-commerce has transformed home deliveries, offering both attended and unattended options. Attended home delivery (AHD) requires the recipient to be present at the time of delivery, which is essential for high-value or perishable goods. In contrast, unattended deliveries allow packages to be left securely without direct handoff.

Subscription-based models (SBMs), such as meal kit services and local produce subscriptions, have gained popularity in e-commerce. These services rely on recurring, scheduled deliveries to customers' homes, making effective management and pricing of delivery time slots crucial. While offering flexible time slot options increases customer convenience, it also introduces logistical challenges. E-retailers must balance customer preferences with operational efficiency to maintain profitability.

Managing time slot availability is critical, as customer choices directly influence delivery routes, costs, and service quality (Nguyen *et al.*, 2019, Klein *et al.*, 2019). Understanding customer preferences enables retailers to optimize time slot selection and pricing, ultimately improving efficiency and customer satisfaction.

This research focuses on the time slot assortment problem in AHD within SBMs. We examine how e-retailers decide which time slots to offer and how to price them while accounting for customer preferences. To better capture heterogeneity in preferences than traditional models, we use a Mixed Logit (ML) model. Given the computational challenges of integrating this model into assortment optimization, we apply simulation-based methods (Pacheco Paneque *et al.*, 2021) to overcome these difficulties.

This study explores both tactical and operational decision-making. At the tactical level, we optimize next-day delivery planning using a profit-maximizing Mixed-Integer Linear Programming (MILP) model. At the operational level, we manage same-day deliveries through a Markov Decision Process (MDP), enabling dynamic, real-time decision-making.

2 Modeling framework

2.1 Retailer Decision Model

In a subscription-based system, an online retailer must determine which delivery time slots and discount rates to offer customers, aiming to maximize profit while considering customers' uncertain preferences.

Let C represent the set of customers, T the set of available time slots, and H the set of possible discount rates, where each $h \in H$ satisfies $0 \leq h < 1$.

The retailer creates an offer set I, formed by pairing each available time slot with each discount rate, as well as an opt-out option ($\{0\}$) that allows customers to decline the offered options. Mathematically, the set I is defined as:

$$I = T \times H \cup \{0\}.$$

For any $i \in I$ (excluding the opt-out option), we represent it as $(t_i, h_i) \in T \times H$, where t_i is a time slot and h_i is a discount rate. The discount is applied to a base fee f, representing the original price before reductions.

The retailer's decision regarding which options to offer is modeled using the binary decision variable γ_{in}, where $\gamma_{in} = 1$ indicates that option $i \in I$ is offered to customer $n \in C$.

2.2 Customer Choice Model

Once the retailer has determined the offer set, customers evaluate the available options. Each customer $n \in C$ faces a set of alternatives I, each characterized by a time slot $t_i \in T$, a discount rate $h_i \in H$, and an opt-out option. The probability that customer n chooses option $i \in I$ depends on the utility they derive from that option, compared to the others.

The utility u_{in} that customer n associates with option i consists of two components: a systematic component V_{in}, which captures observable factors such as time and price, and a random error term ξ_{in}, which accounts for unobserved factors that affect the decision. This can be written as:

$$u_{in} = V_{in} + \xi_{in},$$

where ξ_{in} is assumed to follow an Extreme Value (EV) distribution, which leads to a logit-type probability expression for choice.

The systematic component of the utility V_{in} is modeled as a linear combination of factors that influence customer decisions, such as time, price, and other relevant attributes of the alternatives. Specifically, we assume:

$$V_{in} = f_{\text{time}}(\beta_n^{\text{time}}, t_i) + f_{\text{price}}(\beta_n^{\text{price}}, h_i) + f_{\text{other}}(\beta_n^{\text{other}}, \mathbf{o}_i),$$

where β_n^{time}, β_n^{price}, and β_n^{other} are parameters that capture the customer's preferences for time, price, and other factors, respectively, and $\mathbf{o}_i$ denotes other attributes of alternative i. These parameters reflect how customers value different aspects of the delivery options.

To reflect heterogeneity in customer preferences, we allow some of these sensitivity parameters to vary across individuals. Specifically, we assume that some of the parameters are random, meaning that they are drawn from distributions. This random variation captures the diversity of preferences among customers, ensuring that the model can account for the fact that not all customers value time, price, and other factors in the same way.

Given the presence of these random parameters, the probability that customer n selects option i is no longer simply determined by the deterministic part of the utility V_{in}. Instead, the probability reflects the distribution of these parameters across the customer population. To account for this randomness, the ML model integrates over the distribution of the random parameters β, resulting in the following expression for the choice probability:

$$P_{in}(\mathcal{I}) = \int \frac{e^{V_{in}(\beta)}}{\sum_{j \in \mathcal{I}} e^{V_{jn}(\beta)}} f(\beta) d\beta,$$

where $f(\beta)$ is the probability density function of the random parameters, and $V_{in}(\beta)$ is the utility for option i given the values of the random parameters β.

2.3 Tactical-level time-slot management

Initially, we formulate the tactical-level problem as an MILP model. This model captures the key aspects of time slot assignment, customer choice behavior as represented by the ML model, and delivery routing decisions. The objective function maximizes expected profit while adhering to operational constraints, such as vehicle capacities and time windows. While the MILP formulation provides exact solutions for small-scale instances, it becomes computationally infeasible for larger, real-world scenarios due to the problem's combinatorial complexity and the intricacies introduced by the ML model.

To tackle larger instances, we propose a two-stage metaheuristic approach. The first stage uses a Route-First Time-Second constructive heuristic to efficiently generate initial feasible solutions. This heuristic decomposes the problem into two sequential sub-problems: first, constructing delivery routes based on spatial distribution and vehicle capacities; then assigning time slots to customers along these routes, considering the ML model for customer choice behavior.

Building upon these initial solutions, the second stage employs a simulation-based Adaptive Large Neighborhood Search (sALNS) for solution improvement. This approach combines the advantages of ALNS with Monte Carlo simulation to manage the stochastic nature of customer behavior inherent in the ML model. The sALNS applies various destroy and repair operators to explore the solution space effectively. By incorporating simulation to evaluate the expected revenue of solutions under varying customer choice scenarios, the method provides a robust assessment of solution quality in the face of stochastic customer behavior. This two-stage metaheuristic approach enables us to handle large-scale instances while balancing solution quality and computational efficiency.

To assess the effectiveness of our approach, we conducted numerical experiments on a synthetic dataset, evaluating how well the model optimizes slot assortment while accounting for diverse customer preferences. The results confirm that our sALNS heuristic is particularly well-suited for medium- to large-scale instances. A performance comparison between Gurobi and sALNS (see Table 1a) demonstrates that while Gurobi often struggles with computational limits, sALNS consistently provides high-quality solutions within reasonable time constraints.

Furthermore, Table 1b highlights the advantages of incorporating the ML model over the simpler Multinomial Logit (MNL) model. Our findings indicate that tactical time slot management based on an ML-based choice model, which accounts for heterogeneous customer preferences, consistently performs at least as well as, and often outperforms, tactical time slot management under the MNL assumption, which assumes homogeneous preferences. These advantages are particularly evident in customer populations with high preference heterogeneity. To obtain these results, we solved several instances using the ML- and MNL-based tactical time slot management models, covering various customer distributions and market settings. Each instance was first optimized under its respective choice model. To assess the actual performance of these solutions, we then re-evaluated each optimized solution using a new set of 100 independently generated scenarios. This re-evaluation process allowed us to compare the realized profitability of both models under identical conditions. A series of paired t-tests were conducted on these re-evaluated profit values, confirming that the ML-based model consistently performed at least as well as or better than the MNL-based model in terms of profitability. Even in cases where statistical significance was not reached, the ML-based approach consistently led to higher profitability.

2.4 Operational-level time-slot management

At the operational level, we model a same-day delivery system where the booking and service periods overlap, capturing the dynamic nature of customer arrivals and delivery planning through an MDP. Upon entering the system, the basket value and location of each customer are known, providing key information for delivery planning.

In our MDP, the state $S_t \in \mathcal{S}$ at time t is defined as the combination of all customers currently

Customers	Gurobi		sALNS	
	Time (s)	Gap (%)	Time(s)	Gap with Gurobi (%)
5	16377	0.00	13	1.31
10	43200*	9.98	61	0.21
15	43200*	-	140	-
20	43200*	-	213	-
30	43200*	-	592	-
40	43200*	-	1092	-
50	43200*	-	1892	-
60	43200*	-	3565	-
80	43200*	-	6393	-
100	43200*	-	14468	-

(a) *Performance of sALNS compared to Gurobi, *43200s is the time limit.*

Customers	Map	ML Significantly Better Cases / Total
5	Random	8/10
	Clustered	5/10
	Mixed	9/10
10	Random	3/10
	Clustered	8/10
	Mixed	9/10

(b) *Improvement of ML over MNL in different geographical settings*

in the system, potential future customers, and planned routes. The boundary condition is set as $S_{T+1} = 0$, marking the end of the planning horizon. The probability of customer n arriving at time t is represented by $\lambda_n(t)$. To capture the presence or absence of a request, we introduce the variables ϕ_t^i and ϕ_t^0, respectively.

When a customer arrives and submits a request, our model schedules the delivery using available vehicles at the depot or already on a scheduled tour, accounting for the need to load the customer's demand and return to the depot. The action space of the MDP describes both the time slots offered to the customer and how the routes are updated at each decision point. This dynamic approach allows for real-time optimization of delivery schedules in response to incoming orders. The decision-making process includes key financial components: $r_i^b(\mathcal{I})$, the basket value of option i in the set of offered options $\mathcal{I}$; $r_i^d(t)$, the delivery fee for option i at time t; and $r^l_{\phi^0_{t+1}}$, reflecting logistical rewards at time $t+1$. These components balance immediate revenue from orders with future operational costs.

To formulate the objective function of our MDP, we follow the Bellman equation approach, which is well-established in the literature (Klein & Steinhardt, 2023). This enables us to maximize total profit by accounting for both immediate rewards from customer orders and expected future rewards from optimal decisions. The Bellman equation is given by:

$$V_t(S_t) = \sum_{n \in C} \lambda_n(t) \cdot \max_{\mathcal{I} \in I} \left(\sum_{i \in \mathcal{I}} P_{in}(\mathcal{I}) \left[r_i^b(\mathcal{I}) + r_i^d(t) + r^l_{\phi^0_{t+1}} \right] \right) \tag{1}$$

$$V_t(S_t) = \max_{\mathcal{A} \in \mathcal{A}} E \left[r^b(\mathcal{A}) + V_{t+1}(S_{t+1}) \right], \tag{2}$$

where $\mathcal{A}$ is the action taken at time t, and the objective is to maximize expected rewards by making the best decisions at each step.

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