# First-come-first-served Decentralized Assignment of Capacitated Resources with Partially Observable User Preference

Yuhao Liu[1,2,3], Zhibin Chen *[1,2], Joseph Y. J. Chow[3], and Xi Lin[4]

[1]Shanghai Frontiers Science Center of Artificial Intelligence and Deep Learning, NYU Shanghai, Shanghai, China
[2]Shanghai Key Laboratory of Urban Design and Urban Science, NYU Shanghai, Shanghai, China
[3]C2SMARTER University Transportation Center, Department of Civil and Urban Engineering, New York University Tandon School of Engineering, Brooklyn, NY, United States
[4]Department of Civil and Environmental Engineering, University of Michigan, MI, United States

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## 1 INTRODUCTION

Decentralized assignment systems where heterogeneous users can choose among available alternatives instead of being dictated by some centralized assignment mechanism, play a common role in many real-world applications pertaining to resource distribution such as travelers' route choice and various reservation-based systems (e.g., hotel room booking and ticketing platforms, etc). Predicting/modeling the assignment outcome of such a decentralized system concerns the system manager because the system performance depends on the assignment outcome.

This paper investigates a class of decentralized resource assignment systems where a set of capacitated resources are to be distributed in a first-come-first-served (FCFS) manner to a group of heterogeneous users with partially observable preferences. In particular, different types of resources share some basic functions such that they are considered substitutable. Meanwhile, they are differentiated/classified by features that cater to the preferences of different users. As the users arrive sequentially, each of them can choose among the available types of resources based on their preference. We consider two sources of uncertainty, namely partially observable user preference and unknown arrival order, both of which are motivated by the inadequate information accessibility prevailing in practice.

**Partially observable user preference**. The attributes of alternatives may not be quantifiable or have a presumably consistent preference among all users. To capture the choice behavior of heterogeneous users in these contexts necessitates comprehensive surveys that look into detailed and personalized information about the users. The success of such user behavior studies relies on high-quality data, proper model specification and precise calibration, each of which may be unnecessarily costly and complex (Hensher & Greene, 2003). More importantly, privacy concerns have been an increasingly outstanding obstacle for collecting highly personalized user data. In light of the difficulties in acquiring highly granular user preference information, in this paper, we assume that the manager only has information about users' primary preference (i.e., the most preferred alternative), rather than detailed information that underpins a user behavioral model. The collection of such information can be done in a more practical and credible way using such simpler survey techniques as poll-like micro-surveys that are extremely cost-efficient and user-friendly (Louviere *et al.*, 2000, Hensher, 2006, Dillman *et al.*, 2014).

*Corresponding author.

**Unknown arrival order**. When considering heterogeneous users in a capacitated decentralized assignment system, the capacity constraints plus the FCFS principle render the arrival order pivotal in shaping the assignment outcome (Binder *et al.*, 2017, Paneque *et al.*, 2021). To see this, let's consider a ticketing system where two users make sequential purchases for tickets to a concert. User 1 prefers zone-A tickets, while user 2 prefers zone-B ones. Before their purchases, there is only one zone-B ticket and one zone-C ticket available in the system. Suppose user 1 arrives first. She finds no zone-A tickets left and then chooses to book the only zone-B ticket, leaving the zone-C ticket for user 2. In this case, both users fail to purchase the tickets of their most preferred zones. Now suppose user 2 arrives first, they will definitely book the zone-B ticket and the zone-C ticket will be left for user 1. In most cases, the manager does not have control over users' arrival order. It is therefore necessary for the manager to consider the impact of the unknown arrival order when predicting the assignment outcome. However, existing studies regarding capacitated decentralized assignment systems have not paid enough attention to the subtle relationship between the uncontrolled arrival order and the assignment outcome.

Assignment outcome prediction subject to the aforementioned uncertainty is challenging because the modeler has to precisely determine all possible (and exclude all impossible) scenarios conditioned on the partially known information.

Our work contributes to the literature by filling the aforementioned gaps. First, we establish a utility-free user equilibrium (UE) model, which is the first theoretical result in terms of outcome prediction for a capacitated decentralized assignment system subject to limited knowledge of user preference and with explicit consideration of the FCFS principle for uncontrolled arrival order. Due to the non-uniqueness of UE, we are motivated to assess the best/worst-case UE in terms of user satisfaction, accomplished by solving a mixed integer program with complementarity constraints (MIPCC). Second, we propose a restricted sequential assignment (RSA) framework that serves as an insightful analysis tool for the proposed UE and a bedrock for efficient algorithms for UE-related computations, based on which an efficient dynamic programming (DP) based exact algorithm is designed to solve the MIPCC. Third, the RSA framework and the derived DP enable us to analytically explore the Price of Anarchy (PoA, defined as the ratio of worst-case UE to system optimum) of such decentralized systems. We prove that the PoA in terms of user satisfaction has a tight upper bound $|\mathcal{K}|-1$, where $|\mathcal{K}|$ is the number of resource types. Meanwhile, we also identify a desirable property for the capacity allocation such that the PoA can be controlled to below $|\mathcal{K}|/2$, which signifies huge room for system optimization even subject to uncertainty in user preference and arrival order.

# 2 METHODOLOGY

Consider a reservation-based resource assignment system where finitely many types of capacitated resources (e.g., products, tickets and hotel rooms) are to be reserved/chosen in an FCFS manner by a group of users who arrive sequentially. Let $\mathcal{K}$ be the set of resource types. The supply of type-$k$ resources (or type-$k$ capacity) is denoted by $C^k$ ($k \in \mathcal{K}$). Accordingly, users are divided into $|\mathcal{K}|$ groups based on their primary preference over the resource types; i.e., a user who most prefers type $k \in \mathcal{K}$ resources is categorized as a type-$k$ user. For each type $k \in \mathcal{K}$, the number of type-$k$ users (or type-$k$ demand) is denoted by $Q^k$. Type $k$ is said to be a *surplus type* if $Q^k < C^k$; a *balanced type* if $Q^k = C^k$; a *deficit type* if $Q^k > C^k$. Let $\mathcal{D}$ denote the set of deficit and balanced types and $\mathcal{S}$ that of surplus types.

We use decision variable $x^{kw} \in [0, Q^k]$ to represent the decentralized assignment outcome; i.e., $x^{kw}$ denotes the amount of type-$k$ demand assigned to type-$w$ capacity. For any $k \in \mathcal{K}$, a type-$k$ user is said to be *misplaced* if she is assigned a unit of type-$w$ resources ($\forall w \neq k$). An assignment $\boldsymbol{x} = (x^{kw})_{k,w\in\mathcal{K}}$ is feasible if it satisfies $\sum_{w\in W} x^{kw} = Q^k, \forall k \in \mathcal{K}$, and $\sum_{k\in\mathcal{K}} x^{kw} \leq C^w, \forall w \in \mathcal{K}$. Denote by $\mathcal{X}$ the set of feasible assignments.

Since the assignment is decentralized following the FCFS principle, we need to be able to formally describe the possible assignment outcomes before evaluating the system performance. To this end, we make an assumption about the individual user choice rule.

**Assumption 1. (User choice rule)** *When a type-$k$ user arrives at the reservation system, if there are available type-$k$ resources, then she will reserve one unit of type-$k$ resources. Otherwise, she will* *randomly reserve a unit of available resources regardless of the resource type.*

Note that this is the user choice rule *from the manager's perspective* based on their limited knowledge of user preference for resource types (and facilities), rather than the authentic user choice principle. Nevertheless, the assumption complies with the FCFS principle and individual rationality. Given a sequence of arrivals with each user assumed to make their resource type choice based on the user choice rule, the assignment outcome corresponds to a user equilibrium defined as follows.

**Definition 1. (User Equilibrium, UE)** *A feasible assignment $\boldsymbol{x} \in \mathcal{X}$ is a user equilibrium assignment if it complies with the user choice rule for some sequence of arrivals.*

A UE assignment describes an eventual state that could occur under decentralized assignment from the manager's perspective. It can be shown that it essentially corresponds to a subgame perfect Nash equilibrium. Based on the UE definition, in what follows we mathematically establish the conditions that $\boldsymbol{x}$ should meet so as to be a UE assignment.

**Proposition 1. (UE Condition)** *A feasible assignment $\boldsymbol{x} \in \mathcal{X}$ is a UE assignment if and only if $\exists\, u^k \in \mathbb{R} (\forall k)$ and $v^{kw} \in \{0,1\} (\forall k \neq w)$ such that*

$$0 < \sum_{w \neq k} x^{kw} \perp \left( C^k - \sum_{w} x^{wk} \right) > 0,\ \forall k \in \mathcal{K} \tag{1}$$

$$u^k - u^w \leq (1 - v^{kw})|\mathcal{K}| - 1,\ \forall k \neq w \tag{2}$$

$$x^{kw} \leq Q^k v^{kw},\ \forall k \neq w \tag{3}$$

With Proposition 1, we are now able to capture all UE assignments via an inequality system with complementarity condition (1). Let $\mathcal{X}_{UE}$ denote the set of UE assignments.

It is obvious that the UE assignment is not unique in most cases. In practice, we are particularly interested in how good/bad such a system can be so that we are able to envision the possible outcomes. In this paper, the performance of an assignment $\boldsymbol{x}$ is measured by the manager based on the misplaced demand. The following mathematical program (P1) measures the performance of the best/worst-case UE assignment.

$$\textbf{(P1)}\ \min_{\boldsymbol{x} \in \mathcal{X}_{UE}} / \max_{\boldsymbol{x} \in \mathcal{X}_{UE}}\ z(\boldsymbol{x}) = \sum_{k \in \mathcal{K}} \sum_{w \neq k} x^{kw} \tag{4}$$

Per the UE condition, P1 is intrinsically a mixed-integer program with complementarity constraints (MIPCC). The best-case UE trivially yields $\min_{\boldsymbol{x} \in \mathcal{X}_{UE}} z(\boldsymbol{x}) = \sum_{k \in \mathcal{D}} (Q^k - C^k)$ while the worst-case UE is more difficult to solve. We hence develop a new analysis framework called RSA (omitted due to length limit), based on which P1 (max case) is solved exactly via dynamic programming.

**Proposition 2.** $\max_{\boldsymbol{x} \in \mathcal{X}_{UE}} z(\boldsymbol{x}) = V_{\mathcal{D}}(\mathcal{D}) + V_{\mathcal{S}}(\mathcal{S})$, *where $V_{\mathcal{D}}(\mathcal{D})$ and $V_{\mathcal{S}}(\mathcal{S})$ are the optimal value of following DPs, respectively.*
$V_{\mathcal{D}}(\mathcal{H}) = \max_{k \in \mathcal{H}} \left\{ \min \left\{ \sum_{k \in \mathcal{D} \setminus \mathcal{H}} (Q^k - C^k), C^k \right\} + V_{\mathcal{D}}(\mathcal{H} \setminus \{k\}) \right\}, \forall \mathcal{H} \subset \mathcal{D}$, *and*
$V_{\mathcal{S}}(\mathcal{H}) = \max_{k \in \mathcal{H}} \left\{ \min \left\{ \left[ \sum_{k \in \mathcal{D}} (Q^k - C^k) - \sum_{k \in \mathcal{S} \setminus \mathcal{H}} (C^k - Q^k) \right]^+, C^k \right\} + V_{\mathcal{S}}(\mathcal{H} \setminus \{k\}) \right\}, \forall \mathcal{H} \subset \mathcal{S}$.

Both $V_{\mathcal{D}}(\mathcal{D})$ and $V_{\mathcal{S}}(\mathcal{S})$ can be obtained through backward induction. Numerical experiments suggest that the proposed DP-based solution approach significantly outperforms the solver (CPLEX)-based one. In large-scale instances, the proposed approach can find the globally optimal solution within seconds while the solver-based approach could fail to do that within one hour.

# 3 RESULT

Based on the RSA framework and DPs, we further explore the relation between the system parameters and the worst-case misplaced demand, which offers valuable insights into the implications of the capacity allocation and provides intuitive guidance for user-oriented system design. We measure the efficiency loss due to decentralized assignment using the Price of Anarchy (PoA), defined as the ratio of some performance metric calculated under the worst-case equilibrium to that under the best-case; i.e., $PoA = \frac{V_{\mathcal{D}}(\mathcal{D}) + V_{\mathcal{S}}(\mathcal{S})}{\sum_{k \in \mathcal{D}} (Q^k - C^k)}$.

**Proposition 3. (Bounds on PoA)** *For any system with $|\mathcal{K}| \geq 2$ and at least one deficit type, if the system possesses a unique UE, then $PoA = 1$; otherwise, $1 \leq PoA \leq |\mathcal{K}| - 1$.*

For systems with non-unique UEs, the PoA bounds are tight in the sense that we can vary the PoA exactly from its lower bound to its upper bound merely through leveraging the capacity allocation *regardless of the demand setting and capacity budget.* In other words, capacity allocation has a significant impact on the inefficiency of decentralized assignment. An improper capacity allocation scheme could allow decentralized assignment to generate as much as $|\mathcal{K}| - 1$ times more misplaced demand than centralized assignment. On the contrary, a favorable allocation scheme can do extremely well in reducing the uncertainty due to decentralized assignment. This suggests an enormous potential to investigate the optimal capacity allocation for such decentralized assignment systems subject to ambiguity in user preference and/or arrival sequence.

The following lemma gives bounds on $V_{\mathcal{D}}(\mathcal{D})$ and $V_{\mathcal{S}}(\mathcal{S})$ and offers an intuitive sense of what a robust system (which possesses a lower bound) might look like.

**Lemma 1.** *Let $\overline{k}_1, \overline{k}_2, ..., \overline{k}_{|\mathcal{D}|}$ be a sequence of types in $\mathcal{D}$ such that $\overline{\Delta}_t = Q^{\overline{k}_t} - C^{\overline{k}_t}$ is non-decreasing in $t$, and $\underline{k}_1, \underline{k}_2, ..., \underline{k}_{|\mathcal{S}|}$ a sequence of types in $\mathcal{S}$ such that $\underline{\Delta}_t = C^{\underline{k}_t} - Q^{\underline{k}_t}$ is non-increasing in $t$. Then $V_{\mathcal{D}}(\mathcal{D}) \leq \min\left\{\sum_{t=1}^{|\mathcal{D}|}(|\mathcal{D}| - t)\overline{\Delta}_t, \sum_{k\in\mathcal{D}} C^k - \min_{k\in\mathcal{D}} C^k\right\}$, and $V_{\mathcal{S}}(\mathcal{S}) \leq \min\left\{\sum_{t=2}^{|\mathcal{S}|}\left[\sum_{k\in\mathcal{D}}(Q^k - C^k) - \sum_{t'=1}^{t-1}\underline{\Delta}_{t'}\right]^+, \sum_{k\in\mathcal{S}} Q^k - \min_{k\in\mathcal{S}} Q^k\right\} + \sum_{k\in\mathcal{D}}(Q^k - C^k)$.*

For any given demand profile, suppose that the total capacity for deficit/balanced types is held fixed. $\sum_{t=1}^{|\mathcal{D}|}(|\mathcal{D}| - t)\overline{\Delta}_t$ is minimized if the excess demand $Q^k - C^k$ is equal for all $k \in \mathcal{D}$, while $\sum_{k\in\mathcal{D}} C^k - \min_{k\in\mathcal{D}} C^k$ is minimized if all deficit/balanced types have equal capacity $C^k$. Therefore, either the excess demand or the capacity should ideally be evenly distributed among the deficit/balanced types such that the bound on $V_{\mathcal{D}}(\mathcal{D})$ cannot be further lowered by capacity redistribution among deficit/balanced types. Likewise, the surplus capacity $C^k - Q^k$ should ideally be equal for all $k \in \mathcal{S}$ such that no capacity redistribution among surplus types can improve the bound on $V_{\mathcal{S}}(\mathcal{S})$. We call this a *desirable property* for capacity allocation schemes.

**Proposition 4.** *For any demand profile, the PoA of any capacity allocation scheme observing the desirable property is at most $|\mathcal{K}|/2$.*

Proposition 4 provides a theoretical guarantee for the performance of schemes observing the desirable property. The effect of the property is more prominent when the number of types is large and the upper bound on PoA can be nearly halved. It indicates that despite suffering from limited knowledge of user preference and the lack of operational-level control policy that leverages the arrival/service order and/or the assignment mechanism, there is still a huge room for managing the uncertainty/inefficiency resulted from decentralized assignment via smarter and well-informed system design.

# References


Binder, Stefan, Maknoon, Yousef, & Bierlaire, Michel. 2017. Exogenous priority rules for the capacitated passenger assignment problem. *Transportation Research Part B: Methodological*, **105**, 19–42.

Dillman, Don A, Smyth, Jolene D, & Christian, Leah Melani. 2014. *Internet, phone, mail, and mixed-mode surveys: The tailored design method.* John Wiley & Sons.

Hensher, David A. 2006. How do respondents process stated choice experiments? Attribute consideration under varying information load. *Journal of Applied Econometrics*, **21**(6), 861–878.

Hensher, David A, & Greene, William H. 2003. The mixed logit model: the state of practice. *Transportation*, **30**, 133–176.

Louviere, Jordan J, Hensher, David A, & Swait, Joffre D. 2000. *Stated choice methods: analysis and applications.* Cambridge university press.

Paneque, Meritxell Pacheco, Bierlaire, Michel, Gendron, Bernard, & Azadeh, Shadi Sharif. 2021. Integrating advanced discrete choice models in mixed integer linear optimization. *Transportation Research Part B: Methodological*, **146**, 26–49.