Excess-demand isolation vulnerability analysis based on a bipartitioning minimum cut

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1 INTRODUCTION

Natural disasters such as major earthquakes or severe floods can isolate areas by disrupting access routes. In these circumstances, damage to critical link sets, where disruption of a few connections causes widespread isolation, is of particular concern. Thus, identifying critical link sets and areas vulnerable to isolation is essential for devising proactive disaster-mitigation strategies.

Aerial support can reach isolated areas, but is often delayed. Hence, residents in isolated areas must rely on locally available resources for some periods. A critical isolation scenario arises when resource demand surpasses local supply, causing "excess-demand isolation" and thus shortages. Moreover, substantial excess-demand isolation can precipitate life-threatening conditions. Addressing this, this study proposes an analytical method to determine a set of critical links whose disruption causes excess-demand isolation and to identify isolation-vulnerable areas.

There has been extensive research on connectivity vulnerability (Bell & Iida, 1997, Kurauchi *et al.*, 2009, Sugiura & Chen, 2021). Sugiura & Kurauchi (2023) proposed a minimum cut method to evaluate isolation vulnerability of individual evacuation centers from all designated support bases (i.e., isolation vulnerability of one node). This study extends their work by (1) focusing on scenarios in which multiple nodes are simultaneously isolated from other areas by the disruption of a small number of links (i.e., isolation vulnerability of many nodes), and (2) introducing the concept of excess-demand isolation to identify particularly severe isolation scenarios.

The main contributions of this study are given below.

1. New aspects to aid isolation vulnerability analysis: a) an approach to analyzing isolation vulnerability between many-to-many nodes, and b) the concept of excess-demand isolation.

2. Development of a practical model and a) demonstration that it is solvable with a general optimization solver, b) demonstration of its potential utility in policymaking through an example based on a hypothetical scenario in the Okinawa main road network.

2 METHODOLOGY

2.1 Overview of the method and property of cut

Critical links for excess-demand isolation are found using minimum cuts in bipartitioning directed graphs. A cut between (s,t) is determined as the set of links, $C^{st} \subset E$, such that the graph G is separated into set of nodes, $S, T | s \in S, t \in T, S \bigcup T = V, S \bigcap T = \emptyset$. Among these cuts, the cut consisting of the minimum number of links is denoted the minimum cut and is the most critical for disconnecting the (s,t) pair. However, our approach differs from Sugiura & Kurauchi (2023) as (s,t) pairs are unknown. Instead, we seek to identify links that disconnect subgraphs whose total node weights exceed a threshold value.

2.2 Model derivation

In the following discussion, let $G(V,E)$ be a directed graph, where $i \in V$ and $e \in E$ represent its node and link. In this network, dummy links are denoted as $\bar{e} \in \bar{E}$, and $E + \bar{E} = \dot{E}$. We consider partitioning G into L and R, where $L \cup R = G, L \cap R = \emptyset$. We focus on the situation where R falls into excess-demand isolation, and we regard subgraph R as the isolated subgraph. Each node in G is assigned a weight representing either the demand or supply of resources, denoted as $\psi \in \Psi$. $\psi > 0$ indicates demand value, $\psi < 0$ indicates supply value, and $\psi_i = 0$ indicates that node i has neither demand nor supply. We define $k_i = 0$ if node i is included in subgraph L, and $k_i = 1$ if it is included in subgraph R. For this bipartition, we impose an upper bound K on the number of links included in the cut between subsets L and R. The objective is to maximize excess demand, defined as the sum of node coefficients in one resulting subgraph, subject to the constraint that the number of links in the cut is less than K.

$$\max_{\mathbf{k},\mathbf{l},\theta,\delta} \sum_{i \in V} \psi_i k_i - \epsilon \sum_{e \in E} l_e \tag{1}$$

subject to

$$-k_i + k_j - l_e \leq 0, \forall (i,j) = (j,i) = e \in E \tag{2}$$

$$\sum_{e \in E} l_e \leq K \tag{3}$$

$$\sum_{e \in In(i)} \theta_e - \sum_{e \in Out(i)} \theta_e = \begin{cases} -\sum_{i \in V} k_i & \text{if } i = \bar{s} \\ k_i & \text{otherwise} \end{cases} \tag{4}$$

$$\theta_e \leq M(1 - l_e), \forall e \in E \tag{5}$$

$$\sum_{\bar{e} \in \bar{E}} \delta_{\bar{e}} = 1 \tag{6}$$

$$l_{\bar{e}} \leq M \delta_{\bar{e}}, \forall \bar{e} \in \bar{E} \tag{7}$$

$$k_i = \{0,1\}, \forall i \in V \tag{8}$$

$$l_e = \{0,1\}, \forall e \in E \tag{9}$$

$$\theta_{\dot{e}} = \{0,1,\ldots\}, \forall \dot{e} \in \dot{E} \tag{10}$$

$$\delta_{\bar{e}} = \{0,1\}, \forall \bar{e} \in \bar{E} \tag{11}$$

Eq.(1) maximizes the sum of node coefficients (i.e., the quantity of excess demand) in subgraph R. The term $\epsilon \sum_{e \in E} l_e$, where ϵ is a very small number, ensures selection of the solution with a minimum cut among equivalent optimal solutions. Eq.(2) ensures that for a link e in the cut, $l_e = 1$, and its end nodes satisfy $k_i = 0$ and $k_j = 1$. This constraint holds simultaneously for links in both directions. Eq.(3) imposes an upper limit on the number of links included in the cut between L and R. Eqs.(4) to (7) constrain R to be a single connected component. This ensures identification of the most vulnerable area within the computational domain, as allowing R to consist of multiple geographically distant isolated areas would be inappropriate for this analysis. To enforce this constraint, we introduce an auxiliary network flow, namely $\theta_{\dot{e}}|\dot{e} \in \dot{E}$, which requires the isolated subgraph R to have a tree structure. The process is as follows. (i) Introduce a dummy node connected to all demand nodes via dummy links. (ii) Pass the auxiliary flow of $\sum_{i \in V} k_i$ (equal to the number of nodes in R because $k_i = 1$ for nodes in R) through these dummy links. (iii) Confirm that each node in the isolated subgraph consumes one unit of auxiliary flow. When all auxiliary flow is consumed by the flow conservation constraint, the isolated subgraph forms a single connected component. Additionally, Eq.(5) prevents links with auxiliary flow from overlapping with cut links. Eqs.(6) and (7) jointly constrain auxiliary flow to pass through only one dummy link. This model is an integer linear programming problem and can be solved using general optimization solvers.

3 APPLICATION TO THE OKINAWA ROAD NETWORK

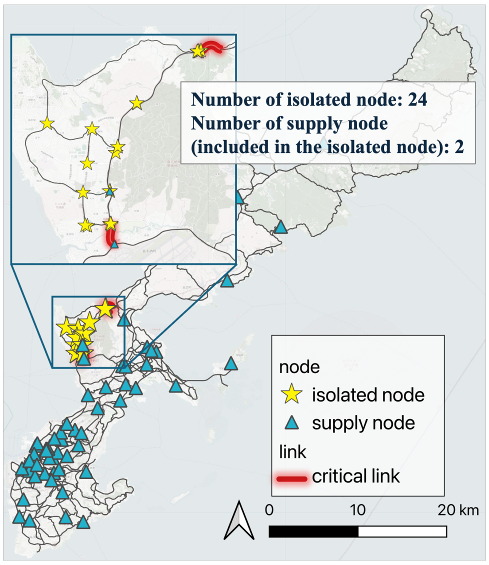
The main Okinawa road network is extracted from OpenStreetMap, yielding a directed network of 1,085 nodes and 2,798 links. All nodes are designated as demand nodes, while supply nodes are identified as those nearest to high schools, i.e., 60 nodes, based on shortest-distance calculations. Node weights are assigned as the local population for demand nodes and 1.01 times the local population for supply nodes, ensuring the entire network does not exhibit excess demand under normal conditions (i.e., when all links are connected). Populations are derived from the intersection of Voronoi polygons centered on each node with the 2020 mesh population data provided by the Ministry of Land, Infrastructure, Transport and Tourism. High schools are chosen as supply nodes due to their wide distribution across Okinawa Prefecture, their representation of local geographic features, and their typical role as disaster prevention centers in Japan.

The number of disconnected links and the calculation results are shown in Figure 1. Scenarios with (a) four critical links (when $K := 4$) and (b) nine critical links (when $K := 9$) are presented. By comparing (a) and (b), which depict scenarios in the same region, it can be seen that as the constraint on the number of disconnected links increases, scenarios that isolate increasingly wide areas are obtained, resulting in increased excess demand. Therefore, if a large earthquake or tsunami causes widespread damage and simultaneously disrupts a certain combination of multiple links in the surrounding area, a wide area may suffer excess-demand isolation, potentially putting many residents in a critical situation. Figure 2 shows the relationship between the constraint on the number of disconnected links and the optimal solution for excess demand. As the graph progresses from left to right, both the optimal solution for excess demand and the number of nodes included in the isolated subgraph increase, showing the same trend as illustrated in Figure 1. The zigzag pattern in the plot of isolated node numbers is due to different regions being included in the isolated subgraph depending on the constraints on the number of disconnected links, resulting in different graph shapes for the isolated subgraph in each scenario.

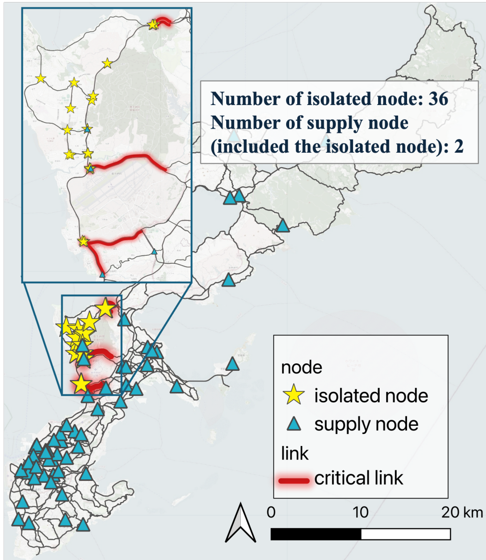
4 DISCUSSION

The solutions identify potentially vulnerable areas within the network, guiding policymakers to prioritize disaster countermeasures according to their budget constraints. Practical implementation requires realistic data, including supplies from both public and private sectors, while considering both residents and visitors. For computational efficiency, network optimization of several thousand links takes minutes on MacOS M3, but larger networks require segmentation based on road characteristics.

Importantly, this methodology can be extended to any infrastructure network that can be represented as a network structure, including power grids, water systems, and supply chains. For



(a) *Scenario of 4 critical links, 24 isolated nodes and 24,278 excess demand cases*



(b) *Scenario of 9 critical links, 36 isolated nodes and 38,644 excess demand cases*

Figure 1 – *Computational examples in Okinawa road network*

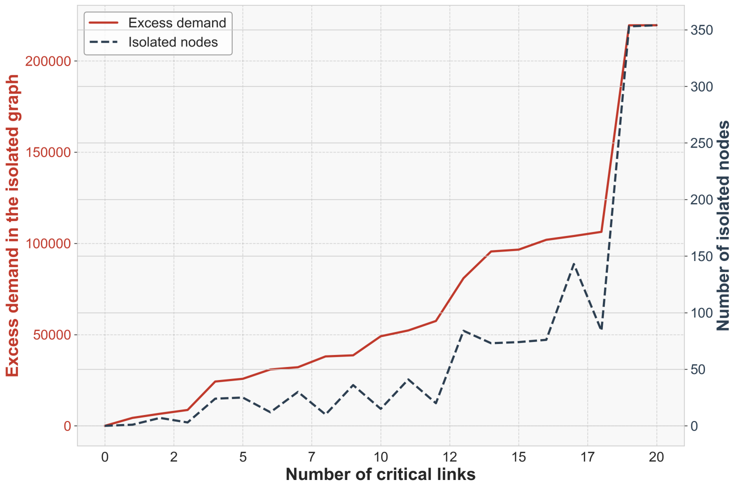


Figure 2 – *Change in isolated nodes and excess demand vs. number of critical links*

future research, solving network design problems to mitigate the identified vulnerabilities would directly support policy planning.

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