Robust planning of bus fleet electrification and charging facility deployment

Yihan Gao[1] and Wei Liu[1]

[1]Department of Aeronautical and Aviation Engineering, The Hong Kong Polytechnic University, Hong Kong, China

Extended abstract submitted for presentation at the 12th Triennial Symposium on Transportation Analysis conference (TRISTAN XII)
June 22-27, 2025, Okinawa, Japan

February 27, 2025

Keywords: electric bus, charging facility, Integer Benders decomposition, Lagrangian relaxation

1 INTRODUCTION

The electrification of transportation system has become a global trend to combat climate change and the usage rate of electric vehicles has been steadily increasing worldwide (Shah & Shah, 2024). It is reported that electric buses are expected to represent 86% of the global fleet as of 2050 (Bloomberg, 2024). The electrification of bus systems can be divided into two parts: i) bus fleet transition that involves determining the electrification sequence and required number of electric buses for each line, and ii) charging facilities deployment that entails identifying optimal locations for charging facilities and specifying the quantity of chargers to be installed. These two parts should be well coordinated to advance the electrification process of bus systems.

Charging supply is the basis for maintaining the regular operation of electric buses. Station-based charging is currently the most popular solution, where charging facilities is usually located at the bus terminal or the dedicated depot (An, 2020, Uslu & Kaya, 2021). Buses of a line requiring charging will stop operations and proceed to designated charging stations, while the remaining buses of this line must be allocated to maintain scheduled services. This may significantly increase the fleet size compared to diesel buses. An optimal configuration of the charging network is crucial for enhancing charging efficiency and saving investment costs. Bus operators also encounter financial challenges. A phased electric bus replacement scheme is more practical, enabling a balance between electrification goals and budgetary constraints.

In the daily operations of electric buses, service frequency and charging requirements typically present uncertainty. The bus service frequency is often limited by the road congestion level. The required charging demand is subject to the road gradient, whether air-conditioning is switched on or off (Doulgeris *et al.*, 2024). This study adopts budget uncertainty sets that allow the flexibility to adjust the conservatism level of robust solutions. Our study makes three main contributions. (i) We propose a research problem that integrates bus fleet electrification and charging facility deployment problems in bus system electrification planning. We consider uncertainties in bus service frequency and charging demand and then formulate the research problem as a mixed integer linear programming model. (ii) We design an exact solution approach that combines Integer Benders decomposition and Lagrangian relaxation methods. Our Lagrangian relaxation method is capable of obtaining a high-quality bound for the subproblem, thereby enhancing the overall solution process. (iii) We conduct numerical studies using data instances from two bus companies in Hong Kong. The results demonstrate that our solution approach can obtain optimal solutions for real-world instances within a practical timeframe.

2 MODEL FORMULATION

This study aims to establish a protracted strategy for implementing bus fleet electrification in an urban public transportation system, considering the bus operator's intention to replace all existing diesel buses with electric buses within a specific planning horizon. We divide the whole planning horizon into $h = 1, 2, \cdots, H$ periods with corresponding budgets. In the beginning of each period, some of the bus lines ($l \in L$) will be electrified, while the charging facility investment needs to follow with the bus electrification process. Specifically, we consider the overall electrification of a bus line as the smallest unit of the bus electrification decision, i.e., the diesel buses of a line will all be (or all not be) replaced with the electric buses in each period h. Table 1 provides a list of decision variables.

Table 1 – *Decision variables used in this study*

Variable	Definition
x_l^h	Binary variable indicating whether bus fleet of line l has been electrified in period h
y_j^h	Binary variable indicating whether charging station j is available in period h
$a_{l,j}^h$	Binary variable indicating whether bus line l is assigned to charging station j in period h
n_j^h	Integer variable denoting the number of chargers at charging station j in period h
b_l^h	Integer variable denoting the number of electric buses of line l in period h
$b_{l,j}^h$	Integer variable denoting the number of electric buses from line l allocated to charging station j in period h

(**Bus fleet electrification constraints**) We need to determine the electrification sequence of bus lines and required number of electric buses. It follows that

$$x_l^h \geq x_l^{h-1}, \forall\, l \in L, h = 1, 2, \cdots, H \tag{1}$$
$$x_l^H = 1, \forall\, l \in L \tag{2}$$
$$x_l^h \leq b_l^h \leq M x_l^h, \forall\, l \in L, h = 1, 2, \cdots, H \tag{3}$$
$$x_l^h \in \{0, 1\}, b_l^h \in \mathbb{Z}_+, \forall\, l \in L, h = 1, 2, \cdots, H \tag{4}$$

Constraints (1) mean that the bus line will always be equipped with electric buses after electrification. Constraints (2) say that all bus lines should be electrified by the end of H. Constraints (3) guarantee that the number of electric buses of line l would equal 0 if this line has not been electrified and greater than or equal 1 otherwise.

(**Charging investment constraints**) The charging investment is specialized by the decisions of charging station location and the number of chargers. It can be established as follows:

$$y_j^h \geq y_j^{h-1}, \forall\, j \in J, h = 1, 2, \cdots, H \tag{5}$$
$$n_j^{h-1} \leq n_j^h \leq M y_j^h, \forall\, j \in J, h = 1, 2, \cdots, H \tag{6}$$
$$n_j^H \leq \mu_j, \forall\, j \in J \tag{7}$$
$$n_j^h \in \mathbb{Z}_+, y_j^h \in \{0, 1\}, \forall\, j \in J, h = 1, 2, \cdots, H \tag{8}$$

Constraints (5) state that the charging stations will always be available after it is constructed. Constraints (6) mean that n_j^h can be greater than 0 only when the charging station j are available in period h, where M is a very large positive constant. Constraints (7) guarantee that the number of chargers installed at charging station j does not exceed the capacity of charging station μ_j.

(**Charging assignment constraints**) We need to assign each electric bus of electrified bus lines to a charging station.

$$b_l^h = \sum_{j \in J} b_{l,j}^h, \forall\, l \in L, h = 1, 2, \cdots, H \tag{9}$$

$$\sum_j a_{l,j}^h \le \delta_l^h, \forall\, l \in L, h = 1, 2, \cdots, H \tag{10}$$

$$a_{l,j}^h \le y_j^h, \forall\, l \in L, j \in J, h = 1, 2, \cdots, H \tag{11}$$

$$a_{l,j}^h \le b_{l,j}^h \le M a_{l,j}^h, \forall\, l \in L, j \in J, h = 1, 2, \cdots, H \tag{12}$$

$$b_{l,j}^h \in \mathbb{Z}_+, a_{l,j}^h \in \{0, 1\}, \forall\, l \in L, j \in J, h = 1, 2, \cdots, H \tag{13}$$

Constraints (9) are the flow conservation of electric buses. Constraints (10) require that the number of charging locations assigned to bus line l cannot exceed a threshold δ_l^h. Constraints (11) stipulate that electric buses can only be assigned to the available charging stations. Constraints (12) state that electric buses can only go to the allocated location for charging.

(**Budget and service level constraints**) The bus fleet electrification process is subject to the given budget and service level. It follows that

$$\alpha_b \cdot \sum_{l \in L} \left(b_l^h - b_l^{h-1} \right) + \alpha_y \cdot \sum_{j \in J} \left(y_j^h - y_j^{h-1} \right) + \alpha_n \cdot \sum_{j \in J} \left(n_j^h - n_j^{h-1} \right) \le \pi^h, \forall\, h = 1, 2, \cdots, H \tag{14}$$

$$\sum_{l \in L} \tilde{\beta}_{l,j}^h b_{l,j}^h \le \zeta_n n_j^h, \forall\, j \in J, h = 1, 2, \cdots, H \tag{15}$$

$$M \cdot \left(1 - x_l^h \right) + \sum_{j \in J} \tilde{\rho}_{l,j}^h b_{l,j}^h \ge \nu_l^h, \forall\, l \in L, h = 1, 2, \cdots, H \tag{16}$$

Constraints (14) request that the total investing cost is restricted within the budget. Constraints (15) guarantee the charging demand should not be exceed its charging supply and constraints (16) say that the bus service frequency should not lower than the required. $\tilde{\beta}_{l,j}^h$ (actual charging requirement) and $\tilde{\rho}_{l,j}^h$ (actual service frequency) are symmetric and bounded random variables with the budget uncertainty set consideration (Bertsimas & Sim, 2003, 2004).

(**Objective function**) The objective function includes investment costs of electric buses, charging stations, and chargers (the first three terms), the emission cost of diesel buses (the fourth term), and the salvage cost of electric buses (the last term).

$$\min \quad \alpha_b \cdot \sum_{l \in L} b_l^H + \alpha_y \cdot \sum_{j \in J} y_j^H + \alpha_n \cdot \sum_{j \in J} n_j^H - \sum_{h=1}^{H} \sum_{l \in L} \alpha_l^h x_l^h - \sum_{h=1}^{H} \sum_{l \in L} \alpha_s^h \cdot \left(b_l^h - b_l^{h-1} \right) \tag{17}$$

3 SOLUTION APPROACH

We solve the problem implementing the Integer Benders decomposition method (Laporte & Louveaux, 1993) adapted to our setting. The master problem includes all binary decision variables and the subproblem includes all integer (not binary) variables and continuous variables. We divide subproblem into several independent small-scale problems using Lagrangian relaxation and decomposition techniques. The objective function values to linear relaxation and Lagrangian relaxation of the subproblem is denoted as Q_{LSP}, Q_{RSP}. Denote κ as an appropriate approximation of the objective function value of subproblem. The solution process is summarized below:

Step 0: Define UB as the upper bound. Initialize the branch-and-bound tree.

Step 1: Select a pendant node. If none exists, stop.

Step 2: Solve the relaxed master problem (RMP) corresponding to the current node. If RMP is infeasible, fathom this node and go to Step 1. Else, let (x, y, a, κ) denote the optimal solution of the current master problem and go to Step 3.

Step 3: Compute Q_{LSP}. If LSP is infeasible, add the feasibility cut, fathom this node and go to step 1. Otherwise, go to step 4.

Step 4: If $\alpha_y \cdot \sum_{j\in J} y_j^H - \sum_{h=1}^{H} \sum_{l\in L} \alpha_l^h x_l^h + \kappa > UB$, fathom this node and go to Step 1. Else, check whether solutions are binary. If all decision variables in RMP are binary, go to Step 5. Otherwise, choose a decision variable violating the binary restriction, create two new branches and append the new nodes to the list of pendant nodes, go to Step 1.

Step 5: If $Q_{LSP}(x, y, a, \kappa) \le \kappa$, go to Step 6. Else, add the linear relaxed optimality cut to the master problem and go to Step 2.

Step 6: Compute $Q_{RSP}(x, y, a, \kappa)$. If RSP is infeasible, add the feasibility cut, fathom this node and go to step 1. Otherwise, if $Q_{RSP}(x, y, a, \kappa) \le \kappa$, go to Step 7, else, add the Lagrangian relaxed optimality cut to the master problem and go to Step 2.

Step 7: Compute $Q(x, y, a, \kappa)$. If $\alpha_y \cdot \sum_{j\in J} y_j^H - \sum_{h=1}^{H} \sum_{l\in L} \alpha_l^h x_l^h + Q(x, y, a, \kappa) < UB$, update $UB = \alpha_y \cdot \sum_{j\in J} y_j^H - \sum_{h=1}^{H} \sum_{l\in L} \alpha_l^h x_l^h + Q(x, y, a, \kappa)$. If $Q(x, y, a, \kappa) \le \kappa$, fathom this node and go to Step 1. Else, generate the integer optimality cut and go to Step 2.

During the algorithm's execution, an exact solution to the subproblem is required only when $Q_{RSP}(x, y, a, \kappa) \le \kappa$. The Lagrangian relaxation optimality cuts are crucial in establishing a high-quality lower bound for the subproblem, which are expected to reduce the computational burden of solving the subproblem exactly. The effectiveness of the algorithm has been verified using data instances from two bus companies in Hong Kong: New Lantao Bus with 36 bus lines and Kowloon Motor Bus (KMB) with 567 bus lines. Table 2 reports the computation results for KMB instance by the standard Integer Benders decomposition method and the proposed method of this study with the different planning horizons. The computation time is set as within 24 hours.

Table 2 – *Computational results from different methods*

Number of periods	Standard Integer Benders decomposition			Proposed method of this study		
	Time (hour)	Gap	Obj	Time (hour)	Gap	Obj
1	2.58	0.00	-2,102,36	0.67	0.00	-2,102,365
5	16.31	0.00	-1,943,208	0.97	0.00	-1,943,208
10	24.00	0.63	-2,250,304	1.23	0.00	-2,634,692
15	-	-	-	2.30	0.00	-2,475,288
20	-	-	-	3.28	0.00	-2,704,491
25	-	-	-	4.13	0.00	-1,000,082
30	-	-	-	5.92	0.00	-593,967

References

An, Kun. 2020. Battery electric bus infrastructure planning under demand uncertainty. *Transportation Research Part C: Emerging Technologies*, **111**, 572–587.

Bertsimas, Dimitris, & Sim, Melvyn. 2003. Robust discrete optimization and network flows. *Mathematical Programming*, **98**(1), 49–71.

Bertsimas, Dimitris, & Sim, Melvyn. 2004. The price of robustness. *Operations Research*, **52**(1), 35–53.

Bloomberg. 2024. *Electric Vehicle Outlook 2024*. Tech. rept. BloombergNEF. Accessed June 12, 2024.

Doulgeris, S, Zafeiriadis, A, Athanasopoulos, N, Tzivelou, N, Michali, ME, Papagianni, S, & Samaras, Z. 2024. Evaluation of energy consumption and electric range of battery electric busses for application to public transportation. *Transportation Engineering*, **15**, 100223.

Laporte, Gilbert, & Louveaux, François V. 1993. The integer L-shaped method for stochastic integer programs with complete recourse. *Operations research Letters*, **13**(3), 133–142.

Shah, Tanjal, & Shah, Manan. 2024. Electrifying the future: Understanding the consumer trends of adoption of electric vehicles in developing nations. *Green Technologies and Sustainability*, 100101.

Uslu, Tugce, & Kaya, Onur. 2021. Location and capacity decisions for electric bus charging stations considering waiting times. *Transportation Research Part D: Transport and Environment*, **90**, 102645.