Multi-layer Network Design for Consolidation-based Transportation Planning

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February 12, 2025

Keywords: Rich Multilayer Network Design, Combinatorial Optimization, Service Network Design, Consolidation-based freight carrier planning

1 INTRODUCTION

Multilayered networks are used to represent complex natural, social, biological, and technological systems, where each layer stands for particular interacting components of the system (Kivelä et al., 2014). A number of Operations Research (OR) network types correspond to this broad definition, e.g., *multi-echelon*, *multi-tier*, and *multilevel* networks. The multilayer network components within any one of these problems correspond to particular network making up a transportation, logistics, or telecommunication system, and interact through transfer links or nodes, providing the means to move flows over multi-component paths.

The OR ***multi-layer network*** term is associated to different transportation and telecommunication settings, in which *an arc in a given layer* is defined with respect to a *set of arcs in another layer* that often make up a path or a cycle. In freight railway planning, for example, a block (group of cars handled together as a unit) is defined, for possible selection in a block layer, in terms of the path of train-service arcs which will transport it if selected in the service layer (Zhu et al., 2014). More than two layers may be involved and the interwoven definitions yield several connectivity relations and requirements in terms of design and flow-distribution decisions, raising challenging *multi-layer network design* (*MLND*) modelling and algorithmic issues. (Crainic, 2024, Crainic et al., 2022).

2 MULTI-LAYER NETWORK DESIGN

Let $\mathcal{L}$ be the set of layers of network $\mathcal{G} = (\mathcal{N}_l, \mathcal{A}_l) = \bigcup_{l\in\mathcal{L}}\{\mathcal{G}_l = (\mathcal{N}_l, \mathcal{A}_l)\}$. The *arc-definition* of the coupled (*supporting*, *supported*) layers $l, l' \in \mathcal{L}$ specifies the set of arcs in the *supporting* layer l that defines an arc in the *supported* layer l'. Figure 1 illustrates the definition, where arcs a, b, and c of the supported layer l' are defined by the sets of arcs (paths, actually) $()\beta, \delta, \epsilon)$, (α, ϵ), and $(\gamma, \delta, \epsilon)$, respectively, in the supporting layer l.

The *connectivity requirements* specify the degree and type of relations between the arc decision variables and the attribute values of $l, l' \in \mathcal{L}$, yielding constraints in the related MLND formulations. The connectivity *degree* indicates whether two or more layers are involved and the direction of involvement: *one-to-one*, illustrated in the figure, *many-to-one*, when several layers support one layer (e.g., trains that may be moved by different engine types), and *one-to-many*, when one layer supports several (e.g., same rail engines supporting passenger and freight trains).

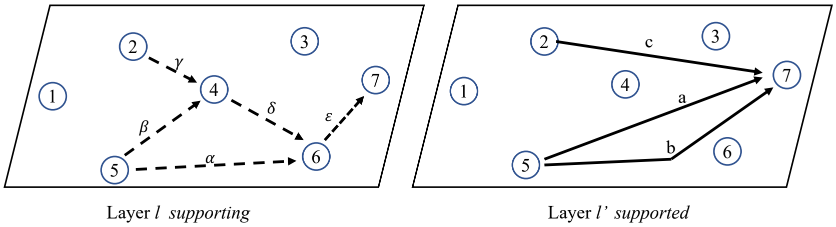

Figure 1: Illustration of arc-definition connectivity

We define three types of layer connectivity relations. The fundamental *design* connectivity constraints enforce existence relations between the selection of the supported and supporting arcs. The *all-design* linking constraints for supported arcs are the most frequently encountered ones, and state that all the supporting arcs must be selected in order for a supported arc to be eligible for selection. *Min-design* linking constraints are introduced when a single supporting arc has to be selected only, in order for the supported arc to be eligible for selection,

Flow-connectivity refers to relations between the flows on the arcs of a coupled layer pair. The fundamental *flow-accumulation* constraint addresses the case when the demand is defined on a supported layer, and states that the commodity flow on a supporting arc equals the sum of that commodity flows on all its supported arcs.

Attribute connectivity has been little studied so far. Generalizing flow connectivity, it concerns the relations between the attributes of the links of the coupled layers, e.g., cost, time, and capacity. The fundamental *supported arc additive-attribute* definition states that the value of an *additive* attribute of a supported-layer arc is given by the sum of the values of the corresponding attributes of the supporting arcs. Distance generally belongs to this class of attributes, as do unit commodity costs and time-related measures. This definition must be verified for the relevant attributes when all the potential arcs on the supporting and supported layers are given as input in the problem setting, e.g., when a potential block in a railway planning application is defined *a priori* as moving on a given sequence of potential train services. The situation is different when the arcs of the supported layer are to be dynamically generated during problem solving, in which case the constraints have to be included in the variable-generation model or procedure.

A different case is observed when addressing arc *capacity*, as feasibility issues have to be addressed. Similar to the discussion above, when the potential supported arcs are pre-defined, verifying that their capacities are not higher than the lowest capacity among the respective supporting arcs guarantees feasibility, together with the all-design linking and the flow-connectivity constraints. The situation is less straightforward when the arc capacities in the supported layer are not known *a priori*, rather belonging to the set of decisions characterizing the problem setting. In such cases, the capacity of a supported arc becomes a decision variable, its “optimal" value to be determined by the interplay among the design decisions in both layers, the capacity of each supporting arc, and the allocation of the latter to all the arcs it supports. The *multi-layer network design problem with capacity decisions* then aims to determine simultaneously the selection of the design arcs on all layers, the arc capacities on the supported layer, and the distribution of demand flows over the resulting multi-layer network, to minimize the total generalized cost of the system. We fully discuss the case within the conference presentation.

The multi-layer network $\mathcal{G}$ is to be designed to satisfy the multicommodity, origin-destination (OD), demand $\mathcal{K}$ (defined on layer $i \in \mathcal{L}$), while accounting for the connectivity relations, as well as for the classic network design constraints. Let d^k be the volume of commodity $k \in \mathcal{K}$ to be moved from its origin $O(k) \in \mathcal{N}_i$ to its destination $D(k) \in \mathcal{N}_i$. Let arcs $a \in \mathcal{A}_l$ be characterized by a fixed cost f_{al}, commodity-specific unit flow costs $c^k_{al}, k \in \mathcal{K}$, and flow capacity u_{al}. A generic multicommodity, fixed-cost, capacitated MLND formulation may be introduced with the following decision variable vectors:

Design $\mathbf{y} = [y_{al}] \in \mathbb{Y}$, where $y_{al} = 1$ if arc $a \in \mathcal{A}_l$ of layer l is selected, 0, otherwise; Alternatively, $y_{al} \in \mathbb{N}$ when the arc may be selected more than once (e.g., the departure frequency of a selected transportation service);

Flow $\mathbf{x} = [x^k_{al}] \in \mathbb{X}$ indicating the quantity of demand $k \in \mathcal{K}$ assigned to arc a of layer l; Depending upon the application, the flow variable may be continuous or integer, but always non-negative.

Let $\mathcal{A}^+_l(i)$ and $\mathcal{A}^-_l(i)$ represent the sets of outgoing and incoming arcs of node $i \in \mathcal{N}_l$, and w^k_i equal d^k if $i = O(k)$, $-d^k$ if $i = D(k)$, and 0, otherwise. The formulation becomes

$$\min \sum_{l\in\mathcal{L}} \left\{ \sum_{a\in\mathcal{A}_l} f_{al} y_{al} + \sum_{k\in\mathcal{K}} \sum_{a\in\mathcal{A}_l} c^k_{al} x^k_{al} \right\} \tag{1}$$

$$\text{s.t.} \sum_{a\in\mathcal{A}^+_l(i)} x^k_{al} - \sum_{a\in\mathcal{A}^-_l(i)} x^k_{al} = w^k_i, \qquad i \in \mathcal{N}_l, k \in \mathcal{K}, l \in \mathcal{L}, \tag{2}$$

$$\sum_{k\in\mathcal{K}} x^k_{al} \le u_{al} y_{al}, \qquad a \in \mathcal{A}_l, \tag{3}$$

$$(\mathbf{y}, \mathbf{x}) \in (\mathcal{Y}, \mathcal{X})_{ll'}, \qquad (l, l') \in \mathcal{C}, l \in \mathcal{L}, \tag{4}$$

$$\mathbf{y} \in \mathbb{Y}, \mathbf{x} \in \mathbb{X}, \tag{5}$$

where the objective function (1) minimizes the total cost of selecting and using arcs on all the layers of the network, while relations (2) and (3) are the typical *flow-conservation* and aggregated *linking capacity* network design constraints. Relations (4) stand for the sets of constraints corresponding to the design, flow, or attribute connectivity requirements proper to the multi-layer network design application at hand.

3 DESIGNING RICH MULTI-LAYER NETWORKS

Rich MLND problems involve more complex arc-definition relations and connectivity requirements compared to the basic two-layer problem setting previously discussed in the literature. Attribute connectivity is often part of rich problem settings, as are arc-definition relations involving more than two layers, and the associated connectivity requirements. Note that, rich MLND applications, particularly present in consolidation-based freight transportation planning, often include several arc definitions and connectivity requirements in the same formulation.

Rich L-layer MLND with $L > 2$ settings may encompass both fundamental two-layer relations, as described above, and generalized many-to-one and one-to-many connectivity definitions and formulations involving more than two layers.

Three general *many-to-one design-connectivity* classes may be encountered when the arcs of a supported layer are defined in terms of several supporting layers: *exclusive*, when at most one of the definitions may be selected (e.g., only one of a given number of ship types (supporting layers) may be assigned to a particular navigation line/service (supported layer).), *required*, when at least an arc must be selected on each of the supporting layers in for the supported arc to be selected (e.g., both traction-power units and crews are required to operate transportation services), and *complementary* indicating the possibility to select more than one definition for a supported-layer arc among its supporting layers, together with the feasible combinations of these definitions (e.g., a freight train service may be defined in terms of three types of locomotives with particular rules on how to assemble the required traction power).

Similar generalization may be defined for flow and attribute-based connectivity. In all cases, additional decision variables and several sets of constraints must be added to MLND formulations to capture the complexity of such rich problem settings.

4 THE TRISTAN PRESENTATION

The presentation first recalls the fundamental MLND concepts. It then details advanced definitions and formulations addressing rich multi-layer networks, with several layers and multi-layer connectivity relations, including the multi-layer network design problem with capacity decisions.

Applications to tactical planning of consolidation-based freight transportation systems illustrate the presentation. Indeed, a very significant part of the transport of goods is performed by consolidation-based carriers, at all geographical scales, from the urban neighborhood, to the region, country, and the world. Less-than-truckload motor carriers, railroads, maritime/coastal/river intermodal carriers, postal and express-courier firms, as well as multi-stakeholder City Logistics, Physical Internet, and synchromodal systems perform consolidation-based services for freight (Crainic et al., 2021). Setting up a profitable and efficient consolidation-based service network is a complex task that requires comprehensive tactical, medium-term, planning. *Scheduled Service Network Design* (*SSND*) is the methodology of choice to support tactical planning (Crainic, 2025a,b, Crainic and Rei, 2024). The problem and methodological challenges became significantly more complex when addressing several sets of design decisions (e.g., vehicles operating different services potentially grouped into platoons), or integrating the management of multiple resources rules by combination and substitution rules (e.g., various intermodal railcar types or vehicles of similar types provided by multiple stakeholders). The associated SSND models are built on time-sensitive multi-layer networks that use advanced layer and connectivity-relation definitions and formulations. The presentation explores the main classes of SSND problem settings and MLND formulations, identifying common characteristics, discussing challenges, and pointing to research avenues.

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