Multimodal stochastic user equilibrium of a tradable credit scheme considering vehicle capacity and passenger waiting time

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1 INTRODUCTION

The growing mismatch between vehicular demand and limited infrastructure has led to severe congestion, calling for innovative solutions to balance demand and capacity. One such solution is the tradable credit scheme, initially designed for emission markets but later adapted for traffic management (Nagurney & Dhanda, 1996, Yang & Wang, 2011). This scheme allocates a fixed number of credits to road users, reducing their disutilities for taking certain alternatives and allowing credit trading for driving privileges or monetary earnings in a regulated market. By enabling this trading, the system keeps traffic within manageable limits, effectively balancing demand with infrastructure capacity, especially during peak periods. Recent studies further expand this scheme with real-time pricing adjustments (Ye & Yang, 2013, Balzer & Leclercq, 2024) and multimodal options(Balzer & Leclercq, 2022, Ding *et al.*, 2023), incentivizing travelers to use public transit, thus reducing congestion and emissions.

Although this scheme effectively reduces the number of vehicles driving on the road, those who lose the option to drive may experience a decline in utility or equity (Elokda *et al.*, 2024). Without adequate alternative transportation, societal welfare might diminish. To mitigate this, we integrate Autonomous Shuttles (AS) into the credit scheme, offering a flexible alternative between private cars and fixed-schedule buses, thus preserving or enhancing overall welfare.

Previous studies modeling transit services have largely overlooked the capacity constraints and the subsequent effect of passenger waiting times on mode choice outcomes. This oversight could misjudge user satisfaction and system efficiency, as waiting times significantly increase the disutility of using transit compared to driving. To address this, we apply a point queue model that dynamically captures passengers' waiting times by considering both capacity limits and the operational schemes of buses and AS over the study horizon. This mechanism integrates with the Macroscopic Fundamental Diagram (MFD) (Mariotte *et al.*, 2017, Lamotte & Geroliminis, 2018) for assessing travel times and incorporates the tradable credit system. The proposed model would serves as a tool to examine how credit incentives influence travel preferences, thereby enhancing traffic flow, optimizing public transit utilization, and improving overall system performance.

2 METHODOLOGY

This study centers on mode choice, using the logit model to capture the probabilistic nature of travelers' decisions among car, bus, and AS options based on each mode's utility. The two main factors influencing these choices are travel time and financial cost. For time-based costs, we apply the trip-based MFD framework, which offers a network-level view of traffic dynamics. For financial costs, we examine the costs and benefits of trading credits.

Within the MFD framework, we consider N groups of travelers with varying origin-destination (OD) pairs. Each group i consists of q_i travelers with a fixed trip length l_i and a specific departure time d_i. The time horizon is discretized into M time slots. For each time slot t, $x(t)$, $y(t)$, and $z(t)$ denote the proportions of travelers using cars, buses, and AS, respectively, during time slot t for OD pair i[1]. The probabilities of choosing car (P_{Car}), bus (P_{Bus}), and AS (P_{AS}):

$$P_{\text{m}} = \frac{\exp(-\theta C_{\text{m}}(t))}{\exp(-\theta C_{\text{Car}}(t)) + \exp(-\theta C_{\text{Bus}}(t)) + \exp(-\theta C_{\text{AS}}(t))}, \quad m \in \{\text{Car, Bus, AS}\} \tag{1}$$

Where, θ is a parameter of the logit model, and $C_{\text{Car}}(t)$, $C_{\text{Bus}}(t)$, and $C_{\text{AS}}(t)$ denote the disutility functions for the car, bus, and AS modes, respectively.

As discussed, the disutility of choosing a mode is driven by two key factors: time and monetary cost. In modeling travel times, we assume that car speed is primarily influenced by the number of vehicles on the road, as cars represent the majority of traffic accumulation. Therefore, the relationship between car travel distance and speed is expressed as:

$$l^{car} = \int_{d}^{d+T^{car}} V^{car}(n^{car}(t))\, dt \tag{2}$$

where, $n^{car}(t)$ denotes the accumulation of cars over time t, $V^{car}(.)$ denotes the speed function depending on the accumulation of cars over time t.

For buses and AS, we initially assume they are operated by the same company, with their travel times adjusted according to peak and off-peak timetables. Thus, the travel time for both modes is simply expressed as $T^{\text{m}} = l^{\text{m}}/V^{\text{m}} \quad m \in \{\text{Bus, AS}\}$, where l^{m} is the trip length and V^{m} is the speed for mode m. However, while the travel time is reliable, waiting time plays a crucial role as it affects the perceived convenience of choosing bus or AS. To accurately describe this, we apply a point queue model to dynamically capture waiting time fluctuations.

Taking bus as an example, we assume that bus passengers arrive uniformly within each time slot t. The accumulated piecewise arrival curve is then defined as $A(t) = A(t_{n-1}) + y(t)q(t)(t - t_{n-1})$, with the initial condition $A(t_0) = 0$. (Refer to "Arrival Curve" in Figure 1 for illustration). Based on the bus schedule and its capacity, C_{Bus}, the departure curve (i.e., the accumulated bus service curve) $S(t)$ is represented as a piecewise constant function: $S(t) = S(t_{n-1}) + \min\{y(t)q(t),\, C_{\text{Bus}}\} \cdot \mathbf{1}(t \geq t_n)$, with the initial condition $S(t_0) = 0$. Here, the number of passengers served increases by $\min\{y(t)q(t),\, C_{\text{Bus}}\}$ at each bus departure time t. (Refer to "Bus Service Curve" in Figure 1 for illustration).

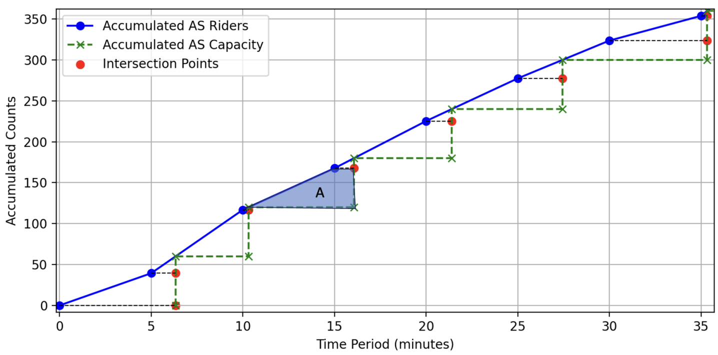

Figure 1 – *Accumulated arrival and bus service curves illustrating passenger waiting times.*

Graphically, the light blue area in Figure 1 serves as an example of cumulative passenger waiting time for the bus between t and $t+1$, and the cumulative waiting time in t can be

[1] Unless otherwise specified, all variables correspond to different OD pairs, and for simplicity, the subscript i will be omitted in the subsequent text.

mathematically expressed as:

$$ACW(t) = \int_{A(t)}^{A(t+1)} t_{service}(\Delta) - t_{arrival}(\Delta)\, d\Delta \tag{3}$$

where $t_{\text{service}}(\Delta)$ is the inverse function of the bus service curve, representing the time when the bus has served Δ passengers, and $t_{\text{arrival}}(\Delta)$ is the inverse function of the arrival curve, representing the time when Δ passengers have arrived. Therefore, we have the average waiting time $W_{\text{bus}}(t)$ for bus rider within time t expressed as $W_{\text{bus}}(t) = ACW(t)/(A(t+1) - A(t))$.

Buses can be viewed as a generalized service, with AS as a specific case. On-demand AS departs at sufficient number of passengers C^p_{AS}, whereas buses can leave partially filled, serving $\min\{y(t)q(t), C_{\text{Bus}}\}$ passengers at each time t.

On the other hand, credits and fares serve as monetary factors influencing travel choices. Travelers receive k credits at the beginning of study period. To promote public transit, car drivers are required to pay τ credits to drive. By combining these monetary costs with time-based costs, such as travel and waiting times, we define disutility functions for cars, buses, and AS as:

$$\begin{cases} C_{Car}(t) = \alpha T(n_{Car}(x(t))) + (\tau - k)p + \delta_{car} \\ C_{Bus}(t) = \alpha T_{Bus} + \alpha' W_{\text{bus}}(y(t)) - kp + \delta_{Bus} \\ C_{AS}(t) = \alpha T_{AS} + \alpha' W_{\text{AS}}(z(t)) - kp + \delta_{AS} \end{cases}, \quad k, \tau \in \mathbb{N},\ \tau \geq k \tag{4}$$

In this expression, α and α' denote the value-of-time (VOT) for time spent inside and outside the transport mode, respectively. The credit price is given by p, while δ_m, where $m \in \{\text{car}, \text{bus}, \text{AS}\}$, represents the fixed cost associated with each mode, such as fuel per mile and ticket fares.

The credit price p is a decision variable determined by the regulated market based on allocation and traveler demand across modes. It is calculated by the formulation $\sum_{t=1}^{T} q(t)k - \sum_{t=1}^{T} q(t)x(t)\tau = 0$ to ensure the Market-Clearing Condition (MCC) (Balzer & Leclercq, 2022), where all allocated credits are consumed by the end of the study period.

Network equilibrium is reached when no traveler has an incentive to switch modes, meaning that the actual mode shares align with the probabilities predicted by the logit model in our game settings. Given the fixed-point nature of the proposed model, we formulate this as a variational inequality (VI) problem (Facchinei, 2003). The vector of car ratios for M time slots is $\mathbf{x} \in [0,1]^M$. Similarly, we define $\mathbf{y}$ and $\mathbf{z}$ as the vectors representing the ratios of bus and AS users, respectively, across all time slots. We further define the function $F(\mathbf{x}, \mathbf{y}, \mathbf{z}, p)$ to capture the equality condition between variable vector and ratio vector

$$F(\mathbf{x}, \mathbf{y}, \mathbf{z}, p) = \begin{pmatrix} \mathbf{x} - \mathbf{P}_{\text{Car}} \\ \mathbf{y} - \mathbf{P}_{\text{Bus}} \\ \mathbf{z} - \mathbf{P}_{\text{CAS}} \\ k - \sum(\mathbf{P}_{\text{Car}} \cdot \tau) \end{pmatrix} \tag{5}$$

Therefore, the VI problem remains to find $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*, p^*) \in K$ bounded by $x, y, z \in [0,1]$, $x + y + z \leq 1$, and $p \geq 0$, such that for all $(x, y, z, p) \in K$:

$$\langle F(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*, p^*), (\mathbf{x}, \mathbf{y}, \mathbf{z}, p) - (\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*, p^*) \rangle \geq 0. \tag{6}$$

3 PRELIMINARY RESULTS

In our numerical study, we apply the model to the A10 motorway segment from Longvilliers to Massy, near Paris. Due to ongoing data collection, we simulate 5,000 travelers' arrival times with a normal distribution centered at 7:30 AM over 36 five-minute intervals between 6:00 and 9:00 AM, focusing first on a peak hour scenario with future plans to extend the model to multiple days. We assume one bus with a 60-passenger capacity per interval and unlimited AS availability, with future plans to consider AS availability limits. Scenarios vary by AS presence and capacity consideration, exploring three combinations of $k = 10$ and $\tau \in \{20, 25, 30\}$.

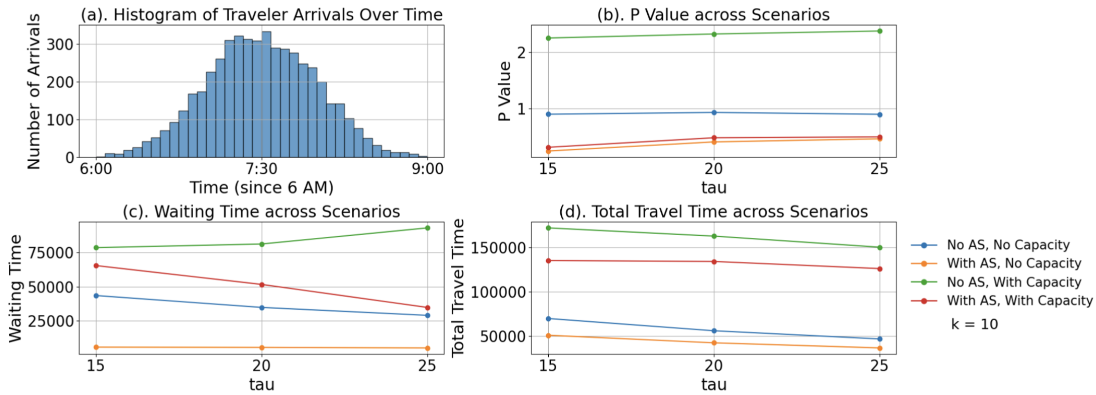

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Figure 2 – *Results Across Different Scenarios.*

4 DISCUSSION

To ensure fair comparison, we incorporate half the headway as waiting time in the bus rider disutility function for scenarios without capacity constraints, following common practice in bus operation studies. As the result shown, capacity constraints and waiting time concerns are critical to consider. Ignoring capacity leads to underestimating the increased waiting times (illustrated in Figure 2(c)) and travel delays (visible in Figure 2(d)), resulting in a misjudgment of the credit system's influence on the choice between public transport and private vehicles. Moreover, due to capacity constraints, increased waiting times encourage travelers to opt for driving, raising the P value (see Figure 2b and Figure 2c)). In this context, introducing AS proves advantageous by reducing both travel and waiting times, effectively redistributing passengers to improve system efficiency and uphold social welfare.

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