Optimizing Skip Schedules for Construction and Demolition Waste Management under Uncertainty

Ze Wang[1], Zhiqi Shao*[2], Michael G.H. Bell[1], D. Glenn Geers[1], and Junbin Gao[2]

[1]Institute of Transport and Logistics, The University of Sydney, Camperdown, NSW 2006, Australia
[2]Business Analytics Discipline, Business School, The University of Sydney, Camperdown, NSW 2006, Australia

February 14, 2025

Keywords: Skip Schedule, Distributionally Robust Optimization, Waste Management

1 Introduction

Skip services play a key role in construction and demolition (C&D) waste logistics. The skip task scheduling problem involves coordinating the delivery of empty skips from a recycling centre to construction or demolition sites, as well as the collection and transport of full skips back to a recycling centre. Since skip rental durations are often multi-day, multi-period planning is required to ensure efficient scheduling. However, challenges such as managing interconnected tasks, balancing vehicle workloads, and addressing travel time uncertainties due to factors like traffic or varying road conditions require robust optimization methods to maximize operational efficiency and ensure timely waste disposal.

Although the skip scheduling problem shares similarities with the Roll-on/Roll-off Vehicle Routing Problem (RR-VRP), the RR-VRP fails to address the unique, multi-period requirements of skip services for C&D waste. Many of these studies also assume vehicles can carry multiple containers and require synchronized pickups and deliveries (Li *et al.*, 2018, Wøhlk & Laporte, 2022). Additionally, previous research focuses on round trips, often neglecting the complex, multi-period task sequences necessary for skip services—such as delivering an empty skip to a job site, collecting the full skip, and transporting it back to the recycling centre.

To address these challenges, we propose a distributionally robust optimization (DRO) framework specifically tailored to skip task scheduling under travel time uncertainty. Unlike existing research, our model is designed to account for the unique operational characteristics of skip services and the inherent variability in travel times. We formulate the problem as a mixed-integer linear programming (MILP) model, aiming to assign tasks while considering constraints such as the number of recycling centres, truck availability, and operating hours. We validate our models using real-world data from Sydney, conducting extensive experiments to evaluate the performance and sensitivity to parameters such as travel time distributions, operating costs, driver constraints, and the number of recycling centres. The results demonstrate that the proposed DRO framework optimizes operational efficiency and supports C&D waste management by effectively addressing the complexities involved in skip task scheduling.

2 Method

The problem focuses on assigning vehicles $\mathcal{M}$ over days $\mathcal{T}$ to service skip demand locations $\mathcal{I}$ and recycling stations (also truck depots) $\mathcal{V}$. Here, $m \in \mathcal{M}$ represents the vehicles, $v \in \mathcal{V}$ the depots or recycling stations, $t \in \mathcal{T}$ the time horizon in days, and $i, j \in \mathcal{I}$ the skip demand locations. The objective is to minimize travel costs while ensuring timely service.

Key decision variables include vehicle schedules $z^t_{v,m}$, with Case 1 representing empty skip delivery $x^t_{i,m,1}$, Case 2 full skip collection $x^t_{i,m,2}$, and Case 3 both delivering an empty skip and collecting a full one $y^t_{i,j,m}$. The skip status $s^2_{i,t}$ is a decision variable that updates based on whether the skip is full and requires collection, while $s^1_{i,t}$ is a parameter that defines the fixed demand for empty skips.

Other parameters include truck usage cost c_m, travel distance costs $d_{i,v}$ and $d_{i,j,v}$, and skip status parameters like the empty skip requirement $s^1_{i,t}$. These elements are incorporated to minimize total costs while ensuring all service requirements are met. The objective function is as follows:

*Corresponding author. Email: zhiqi.shao@sydney.edu.au

$$\min \sum_{m,t} c_m f_{m,t} + \sum_{m,t} \left(\sum_{v,i} d_{i,v}(x^t_{i,m,1} + x^t_{i,m,2}) z^t_{v,m} + \sum_{v,i,j} d_{i,j,v} y^t_{i,j,m} z^t_{v,m} \right) \tag{1}$$

In this equation, c_m represents the cost of deploying vehicle m on any given day t, while $f_{m,t}$ is a binary decision variable that equals 1 if vehicle m is deployed on day t, and 0 otherwise. Furthermore, travel cost $d_{i,v}$ between skip demand location i and recycling center v, and it accounts for both empty skip deliveries ($x^t_{i,m,1}$) and full skip collections ($x^t_{i,m,2}$) by vehicle m on day t. Additionally, $y^t_{i,j,m}$ denotes the inter-skip trips, where a vehicle moves between skip demand locations i and j. The vehicle utilization constraint is to ensure that vehicles can only be assigned to recycling centers if they are actively in use on that day:

$$z^t_{v,m} \leq f_{m,t} \quad \forall v \in \mathcal{V}, m \in \mathcal{M}, t \in \mathcal{T}. \tag{2}$$

Here, $z^t_{v,m}$ is a binary variable that equals 1 if vehicle m serves recycling center v on day t, and 0 otherwise. This ensures that a vehicle can only be assigned to a recycling center if it is operational, as indicated by $f_{m,t} = 1$. A further constraint ensures that vehicle $m+1$ can only be deployed if vehicle m is already in use. This sequential vehicle deployment is necessary to maintain operational structure and is expressed as:

$$f_{m+1,t} \leq f_{m,t} \quad \forall m \in \mathcal{M}, t \in \mathcal{T}, \tag{3}$$

which helps enforce a logical deployment of vehicles, where vehicles are deployed in a specific order, preventing vehicle $m+1$ from being used unless vehicle m is already operating.

The model includes a constraint on the number of skips that a vehicle can service a day:

$$\sum_{i \in \mathcal{I}} (x^t_{i,m,1} + x^t_{i,m,2}) + \sum_{i,j \in \mathcal{I}} y^t_{i,j,m} \leq M \cdot z^t_{v,m} \quad \forall v \in \mathcal{V}, t \in \mathcal{T} \tag{4}$$

Where, the terms $x^t_{i,m,1}$ and $x^t_{i,m,2}$ represent the binary decision variables for delivering empty skips to, or collecting full skips from, location i by vehicle m on day t. The term $y^t_{i,j,m}$ captures the trips between skip locations i and j. The constraint ensures that the total number of tasks assigned to a vehicle does not exceed its capacity, with M representing the maximum capacity of vehicle m.

To ensure that a recycling center can be assigned with at most one vehicle:

$$\sum_{m \in \mathcal{M}} z^t_{v,m} \leq 1 \quad \forall v \in \mathcal{V}, t \in \mathcal{T} \tag{5}$$

The sum of the delivery, collection, and inter-skip trip binary variables is limited to 1, ensuring that a vehicle can perform only one task at a time:

$$\sum_{v \in \mathcal{V}} \left(x^t_{i,m,1} + x^t_{i,m,2} + \sum_{j \in \mathcal{I}} y^t_{i,j,m} \right) \leq 1 \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{6}$$

The following constraint manages the demand for empty skips at each location. It guarantees that a vehicle is assigned to deliver an empty skip to the specified location on the required day:

$$\sum_{v \in \mathcal{V}} x^t_{i,m,1} + \sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{I}} y^t_{i,j,m} \leq s^1_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{7}$$

$x^t_{i,m,1}$ is a binary variable indicating whether vehicle m delivers an empty skip to location i on day t. $y^t_{i,j,m}$ represents the inter-skip trips. $s^1_{i,t}$, is also a binary parameter that equals 1 if demand location i requires an empty skip on day t, and 0 otherwise.

Similarly, the demand for collecting full skips is captured by the following constraint:

$$\sum_{v \in \mathcal{V}} x^t_{i,m,2} + \sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{I}} y^t_{j,i,m} \leq s^2_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{8}$$

$x^t_{i,m,2}$ is the binary variable representing whether vehicle m collects a full skip from location i on day t, while $y^t_{j,i,m}$ represents an inter-skip trip that also involves a full skip collection. The parameter $s^2_{i,t}$ is 1 if skip i is full and requires collection on day t, and 0 otherwise. It ensures that skips are collected promptly.

In our study, we assume that an empty bin should be sent to the demand point within a time interval $\mathcal{T}^i_k$, where delivery delays are unacceptable, with the constraint defined as:

$$\sum_{t \in \mathcal{T}^i_k} \left(\sum_{v \in \mathcal{V}} x^t_{i,m,1} + \sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{I}} y^t_{i,j,m} \right) = 1 \quad \forall i \in \mathcal{I}, k \in \{1, \ldots, K_i\} \tag{9}$$

$\mathcal{T}_k^i$ represents the set of days within period k for skip demand location i, and K_i denotes the set of days for requiring empty skip for location i e.g, $\mathcal{T}_k^i = \{i : \{\text{day}1, \ldots, \text{day}k\}\}$. Empty skips are required to be delivered within days in $\mathcal{T}_k^i$. The sum ensures that skip i receives exactly one empty skip delivery within each period k, guaranteeing that the schedule is adhered to, preventing both missed and redundant deliveries.

To account for the skip rental period at location i. This constraint ensures that the need for collecting full skips is recognized after the skip has been in place for a sufficient number of days:

$$s_{i,t}^2 = \sum_{v \in \mathcal{V}} \left(\sum_{j \in \mathcal{I}} y_{i,j,m}^{t-a_i} + x_{i,m,1}^{t-a_i} \right) \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{10}$$

Here, a_i is the skip rental period for at location i in days. The constraint links the need for a full skip collection ($s_{i,t}^2 = 1$) to the delivery of an empty skip a_i days prior.

Finally, the following constraint ensures that once a skip is full, it is collected promptly:

$$\sum_{v \in \mathcal{V}} \left(x_{i,m,2}^t + \sum_{j \in \mathcal{I}} y_{j,i,m}^t \right) \geq s_{i,t}^2 \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{11}$$

The left-hand side represents the sum of full skip collections ($x_{i,m,2}^t$) and inter-skip trips involving full skips ($y_{j,i,m}^t$), and the right-hand side, $s_{i,t}^2$, indicates whether skip i is full on day t. Ensures that if a skip is full, it is either collected or included in a scheduled trip between skip demand locations.

To handle uncertainty in travel times, the model introduces the random variable $\tilde{\tau}_{i,j}$, representing uncertain travel times between locations i and j, with specific scenarios $\tau_{i,j}^s$ for each sample s. This ensures the schedule remains feasible under varying conditions. The robustness model adjusts travel times using parameters like the mean $\mu_{i,j}$, upper bound $\overline{\tau}_{i,j}$, lower bound $\underline{\tau}_{i,j}$, and standard deviation $\sigma_{i,j}$, applying a conservatism factor ϵ to control how cautiously variability is handled. The adjusted travel time $\tau'_{i,j}$ represents the worst-case scenario. Uncertainty is managed through a distributionally robust optimization (DRO) approach, assuming travel times belong to an ambiguity set $\mathcal{P}$, defined by their mean $\mu_{i,j}$ and variability $\sigma_{i,j}$:

$$\mathcal{P} = \{\mathbb{P} \in \mathcal{P}_0(\mathbb{R}^n) : \mathbb{E}_{\mathbb{P}}[\boldsymbol{\tau}] = \boldsymbol{\mu}, \quad \mathbb{E}_{\mathbb{P}}[|\boldsymbol{\tau} - \boldsymbol{\mu}|] \leq \boldsymbol{\sigma}\} \tag{12}$$

Here, $\mathbb{P}$ represents a probability distribution over travel times $\boldsymbol{\tau}$, and $\mathbb{E}_{\mathbb{P}}[\cdot]$ denotes the expected value. The ambiguity set $\mathcal{P}$ is defined using the mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$ to capture both the expected value and variability of travel times. This ensures robustness without assuming a specific distribution, protecting against worst-case scenarios. The model includes a chance constraint to ensure that, with high probability, the total travel time in any region R_k does not exceed a specified limit D:

$$\mathbb{P}\left[\sum_{i \in R_k} f(x_i, y_i, z_i) \leq D\right] \geq 1 - \epsilon \quad \forall \mathbb{P} \in \mathcal{P} \tag{13}$$

It ensures that, under any probability distribution in the ambiguity set $\mathcal{P}$, the total travel time for all tasks assigned in region R_k does not exceed the maximum allowable time D with high probability. The parameter ϵ controls the level of conservatism in the model, with smaller values of ϵ corresponding to more conservative solutions. The chance constraint guarantees that the system operates within acceptable limits in most scenarios, thereby reducing the risk of schedule violations due to delays.

To account for the worst-case travel time, the model uses the Probability Value-at-Risk (P-VaR) at confidence level $1 - \epsilon$. The worst-case scenario is modeled as:

$$\sup_{\mathbb{P} \in \mathcal{P}} \text{P-VaR}_{1-\epsilon}\left[\sum_{i \in R_k} f(x_i, y_i, z_i)\right] \leq D \tag{14}$$

In this formulation, the sup operator takes the supremum (or the worst-case value) over all distributions $\mathbb{P}$ in the ambiguity set $\mathcal{P}$, and $\text{P-VaR}_{1-\epsilon}$ represents the probability value-at-risk at confidence level $1 - \epsilon$. This ensures that even in the most adverse travel time scenarios, the total travel time for the tasks remains within the allowable limit D. This risk-averse approach is essential in situations where uncertainty in travel times can significantly impact the operational efficiency of the system.

The adjusted travel time $\tau'_{i,j}$ is computed based on the worst-case scenario, ensuring robustness. It is calculated as:

$$\tau'_{i,j} := \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{P}\text{-VaR}_{1-\epsilon}[\tilde{\tau}_{i,j}] = \mu_{i,j} + \min\left\{\overline{\tau}_{i,j} - \mu_{i,j}, \frac{1-\epsilon}{\epsilon}(\mu_{i,j} - \underline{\tau}_{i,j}), \frac{1}{2\epsilon}\sigma_{i,j}\right\} \tag{15}$$

Here, $\mu_{i,j}$ is the mean travel time between locations i and j, $\overline{\tau}_{i,j}$ and $\underline{\tau}_{i,j}$ are the upper and lower bounds on travel time, and $\sigma_{i,j}$ is the standard deviation of the travel time. The equation adjusts the travel time by considering the worst-case scenario, ensuring that the model accounts for uncertainty in a conservative yet realistic manner. The conservatism parameter ϵ influences the adjustment, with smaller values of ϵ resulting in more cautious adjustments. Finally, the total adjusted travel time across all vehicle tasks must stay within the daily limit D, even in the worst-case scenario:

$$\sum_{v\in\mathcal{V}}\sum_{t\in\mathcal{T}}\tau'_{i,v}x^t_{i,m,1}z^t_{v,m}+\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}}\tau'_{i,v}x^t_{i,m,2}z^t_{v,m}+\sum_{v\in\mathcal{V}}\sum_{i,j\in\mathcal{I}}\tau'_{i,j,v}y^t_{i,j,m}z^t_{v,m}\leq D, \tag{16}$$

where, the first term represents the total adjusted travel time for empty skip deliveries ($x^t_{i,m,1}$) from recycling centers v to skip locations i, the second term accounts for the travel time for full skip collections ($x^t_{i,m,2}$), and the third term captures the travel time for inter-skip trips ($y^t_{i,j,m}$) between locations i and j. This constraint ensures that all vehicle assignments' total adjusted travel time remains within the daily limit D, even under worst-case travel time scenarios.

3 Results & Discussion

The model covers a 14-day period across 33 Local Government Area locations in Sydney as skip demand points, with four recycling sites serving as truck depots. Skip demand is population-based, and each empty skip is collected three days after delivery. A fixed truck usage cost of 5 hours of driving cost applies, with a maximum of 8 hours of driving allowed per day.

Parameters		**Results**		
Condition	**Model**	**Best objective(h)**	**Time violation**	**Computation Time (s)**
$\mathcal{P}=0.7$	Baseline	391	10.00%	207.50
	Ours	399	1.40%	87.74
$\mathcal{P}=0.9$	Baseline	391	6.82%	274.39
	Ours	412	0.15%	263.53
Max travel hour	Baseline	403	11.69%	275.03
($D=7$)	Ours	433	1.02%	262.98
Max travel hour	Baseline	378	9.48%	208.31
($D=9$)	Ours	391	0.76%	121.38
Truck fix cost	Baseline	299	9.01%	594.58
($c_m=2.5$)	Ours	303	0.76%	127.16
One depot ($V=1$)	Baseline	546	21.11%	14.69
	Ours	564	3.64%	3.04
Two depots ($V=2$)	Baseline	449	17.11%	39.13
	Ours	479	1.60%	11.22
Three depots ($V=3$)	Baseline	393	9.01%	78.91
	Ours	410	0.67%	34.19
Four depots ($V=4$)	Baseline	391	8.69%	268.99
	Ours	403	0.70%	160.49

Table 1 – *Comparison of Baseline and Our Model under Various Conditions*

In Table 1, the baseline refers to our skip task assignment model using a standard DRO approach, while the proposed model incorporates our novel DRO method into the skip task assignment framework. The comparison evaluates the performance under various conditions, including different values of $\mathcal{P}$ (the probabilistic guarantee against workload violations), as well as factors such as travel hours, truck costs, and depots. Increasing $\mathcal{P}$ leads to higher objective costs by reducing the allowance for violations. While both models achieve optimal solutions, our proposed model exhibits fewer time violations and faster computation times, indicating superior handling of uncertainty and overall improved efficiency.

References

Li, Hongqi, Jian, Xiaorong, Chang, Xinyu, & Lu, Yingrong. 2018. The generalized rollon-rolloff vehicle routing problem and savings-based algorithm. *Transportation Research Part B: Methodological*, **113**, 1–23.

Wøhlk, Sanne, & Laporte, Gilbert. 2022. Transport of skips between recycling centers and treatment facilities. *Computers & Operations Research*, **145**, 105879.